ISSN: 0974-2115
www.jchps.com Journal of Chemical and Pharmaceutical Sciences

# Investigation of Electron Scattering Calculations for $^{17,18}$O and $^{20}$Ne Using Extended Model Space

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**ABSTRACT**

The shell model calculations in the present work are performed using NuShell@MSU code for windows without any restriction imposed on the model space within large-scale sdpf-model space. Elastic and Inelastic electron scattering form factors , reduced transition probabilities and the charge density distribution have been calculate based on the sdpfnow effective interaction and compared the results with the experimental data . The radial wave function for the single-particle matrix elements have been calculated with the Harmonic Oscillator (HO) and Skyrme (SKX) potentials in the model space and Tassie model for $^{17,18}$O and $^{20}$Ne nucleus.

**KEY WORDS**: shell model, (e, e) inelastic longitudinal form factors, Harmonic Oscillator and Skyrme potentials, core-polarization effects, NuShell, Nuclear Structure.

## 1. INTRODUCTION

Electron scattering provides information about the nuclear structure include size, charge distribution and the electromagnetic currents inside the nuclei. Theoretical work on electron scattering starts from 1929, when mott derived the cross section for the relativistic scattering of Dirac particles (Mott, 1929). The nuclear size can be taken into account by multiplying the mott cross section by the form factor which depends on the charge, current and magnetization distribution of the nucleus. Modern, large-basis shell-model calculations are progressing in several dimensions: (a) better methods and models to predict which of the basis states are most important , so that the reach of the active space can be extended without proportional expansion of dimensionalities; (b) improved mathematical and computational techniques for projecting angular momentum and setting up and diagonalizing large matrices; (c) use of less restrictive , but more discriminating and realistic, assumptions about the effective nucleon-nucleon interaction; and (d) theoretical and empirical elucidation of the effects of excluded configurations (Brown and Wildenthal, 1988) . These developments are proceeding in parallel with the use of computers that are faster and have larger memories. Advances on these fronts rely on the contributions of many groups working with several different approaches on several different regions of nuclei (Brown, and Wildenthal, 1988). The sdpf-shell is considered in the present work, this model deals with the distribution and coupling of the valence nucleons within an extended model-space. According to this model the $^{16}$O is considered as an inert core. The configuration mixing nuclear shell model allows the mixing of different orbits to create the eigenstates, this model still assumed that the nucleus contain an inert core and active orbits in which the valance nucleons are distributed according to Pauli principle. In this case, all the valance nucleons share in the scattering process. The sdpf-shell model calculations consider only the role of sdpf-shell. The inelastic electron scattering calculations take the role of the core into account by using effective charges or effective $g$-factors. Some theoretical results of nuclear structure for many light nuclei in p and sd shells have been discussed (Jassim, 2012; Jassim and Al-Sammarrae, 2014; Jassim, and Rawaa A.Abdul-Nabe, 2016; Jassim, 2016) using shell model calculations. A large-scale shell model calculation also performed for Nuclear Structure of $^{104,106,108}$Sn Isotopes (Jassim, 2013) with the Sn100pn interaction. Theoretical results of nuclear structure for many $^{17}$O nuclei in sd shells have been discussed using shell model calculations (Khalid Jassim, 2013; Majeed and Najim, 2015).

The aim of the present work is to study various components of the electron scattering form factors, charge density distributions and reduced transition probabilities for $^{17,18}$O and $^{20}$Ne by means of large-scale shell model calculations without any restrictions by employing sdpfnow effective interaction and compare the theoretical results with experimental data.

## 2. METHODS & MATERIALS

**Theory:** The elastic electron scattering form factor from is simply the Fourier transform of the charge density distributions $\rho_{ch}(r)$ , given by Wong (2008):

$$F^L(q) = \int \rho_{ch}(r) e^{iqr}\, dV \qquad (1)$$

$F^L(q)$ is known as the longitudinal form factor . The electron scattering form factor with the corrections, in terms of angular momentum J and momentum transfer q, and inclode isospin, can be written as (Donnelly, 1984)

$$\left|F_J^\eta(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\sum_{T=0,1}\begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z\end{pmatrix}\langle f\|\|\hat{T}_{JT}^\eta(q)\|\|i\rangle\, F_{cm}(q)\, F_{fs}(q)\right|^2 \quad (2)$$

Where η indicate the longitudinal (C), transverse electric (E), and transverse magnetic (M) form factors. $\hat{T}_J^{\eta}(q)$ is the electron scattering operator.

$F_J^{\eta}(q) = e^{q^2 b^2/4A}$ is the corrections of the Center-of-mass, A and b are the mass number and the harmonic oscillator size parameter, respectively, and $F_{fs}(q) = e^{-0.43q^2/4}$ is the corrections of nucleon finite-size. We can rewrite the nuclear many-body matrix elements in terms of the one-body matrix element and the reduced one-body matrix element (Jassim, 2014; Brussaard, 1977).

$$\langle f|||\hat{T}_{JT}^{\eta}|||i\rangle = \sum_{ab} OBDM(i,f)\,\langle j_f|||\hat{T}_{JT}^{\eta}|||j_i\rangle \qquad (3)$$

Where quantum numbers (n, $l$, j) abbreviated by j. The reduced single-particle matrix element in both spin and isospin, can be rewritten in terms of the single particle matrix element reduced in spin only (Donnelly, 1984; Brussaard, 1977).

$$\langle j_f|||\hat{T}_{JT}^{\eta}|||j_i\rangle = \sqrt{\frac{2\pi+1}{2}} \sum_{\tau_z} I_T(\tau_z) \left\langle j_f||\hat{T}_{J\tau_z}^{\eta}(q)||j_i\right\rangle \qquad (4)$$

where

$$I_T(\tau_z) = \begin{bmatrix} (-1)^{-t_z+\frac{1}{2}}, & for\ T=1 \\ 1, & for\ T=0 \end{bmatrix} \qquad (5)$$

And $t_z$ = 1/2 and −1/2 for the proton and neutron, respectively. In the present work, the shape of the Tassie Model is employed for core polarization. The effect of core polarization is found to be essential for both the transition strengths and the momentum-transfer dependence and gives a good description of the data (Jassim, 2014). The longitudinal form factors for this model are (Tassie, 1956).

$$F_J^L(q) = \sqrt{\frac{4\pi}{2J_i+1}}\ \frac{1}{Z}\left[\int_0^{\infty} r^2 j_J(qr) \overset{ms}{\rho}_{Jt_z} dr - Nq\int_0^{\infty} dr\, r^{J+1}\rho_o\, j_{J-1}(qr)\right] \times F_{cm}(q)\, F_{fs}(q) \qquad (6)$$

Where N is a proportionality constant and $\rho_o$ is the ground state two – body charge density distribution, and j is the spherical Bessel function. The reduced electric transition strength is given by (Brown, 1985).

$$B(EJ) = \frac{Z^2}{4\pi}\left(\frac{(2J+1)!!}{K^J}\right)^2 \left|F_J^L(k)\right|^2, \qquad (7)$$

Where $k = \frac{E_x}{\hbar c}$

**3. RESULTS AND DISCUSSIONS**

In this work, we are calculated various components of electron scattering form factors for the $^{17,18}O$ and $^{20}Ne$ nuclei, which have ground state spin-parity $J^{\pi}$ = 5/2$^+$, 0$^+$, 0$^+$, respectively. Sdpf-extended model space has been adopted in order to distribute the valence particles outside an inert core $^{16}O$. The active orbits of this model space are $1d_{5/2}\,2s_{1/2}\,1d_{3/2}\,1f_{7/2}\,2p_{3/2}\,1f_{5/2}\,2p_{1/2}$. The radial wave functions for the single-particle matrix elements were calculated with the harmonic oscillator (HO) and Skyrme potentials. The oscillator length parameter b are at 1.763 fm, 1.821 fm, 1.869 fm for $^{17}O$, $^{18}O$ and $^{20}Ne$ nucleus, respectively. We employed the effective charge 0.35 for each of proton and neutron for the $^{20}Ne$ nucleus which have two protons and two neutrons in the model space. The neutron effective charge is 0.65 for $^{17,18}O$ nucleus, where the model space has only neutrons. Fig.1, shows the longitudinal C2 ($1/2_1^+$ 1/2) inelastic electron scattering form factor as a function of momentum transfer q in the $^{17}O$ nucleus. Only Tassie calculations give a good agreement comparing with the experimental data (Manley, 1987). The calculations of model space without effective charge are very small compared with the experimental data. In fig.2, we show the transverse M3 form factor for this state. The HO calculations give a good agreement with the experimental data (Manley, 1987). The model space with Skyrme potential calculations are less efficiency in this case.

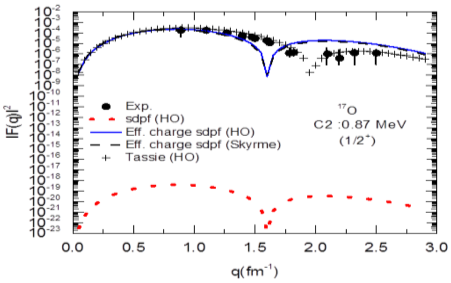


**Figure.1. The longitudinal C2 form factors for the ($1/2_1^+$ 1/2) (0.87 MeV) in the $^{17}$O nucleus**

Experimental values are indicated by the filled circles (Manley, 1987). The dotted and solid curves indicate the calculations of HO potential without and with effective charge, respectively. The dashed curves indicate the calculations of Skyrme potential with effective charge. The "+" symbols indicate the calculations of TM.

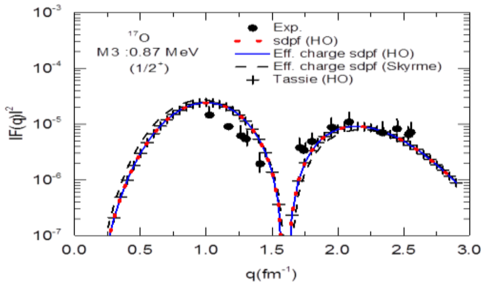


**Figure.2. The transverse M3 form factors for the $(1/2_1^+\ 1/2)$ (0.87 MeV) in the $^{17}O$ nucleus**

Experimental values are indicated by the filled circles (Manley, 1987). The longitudinal C0 $(0_1^+\ 1)$ elastic electron scattering form factor in the $^{18}O$ nucleus is shown in Fig.3. The TM calculations give a poor agreement with the experimental data (Kuchta, 1988). The calculations of model space without effective charge give a good agreement with the experimental data. In such cases, we note the difference of effects between the effective charge model space and TM (core-polarization) calculations for ground states. The calculations of model space without effective charge for C2 $(2_1^+\ 1)$ form factor in the $^{18}O$ nucleus again less than the experimental data (Kuchta, 1988) as shown in Fig.4, while the calculations of model space with effective charge give a poor agreement with the experimental data. Only TM calculations give a good agreement with the experimental data. The effective charge for proton is not affected for $^{17,18}O$ nucleus when we calculate the longitudinal form factors because the active nucleons are only one and two neutrons, respectively.

Fig.5, shows the longitudinal C5 $(5_3^-\ 0)$ inelastic electron scattering form factor in the $^{18}O$ nucleus. Again the calculations of model space without effective charge are very small compared with the experimental data, that may be because it gives zero effective charge for neutron. We note that the calculations of model space with HO and Skyrme which consist of one peak for excited states in the $^{18}O$ nucleus.

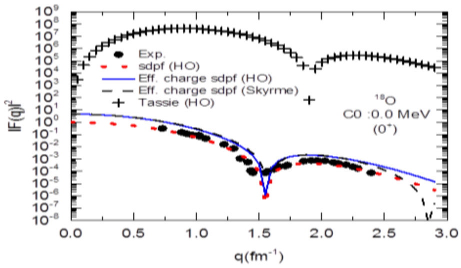


**Figure.3. The longitudinal C0 form factors for the $(0_1^+\ 1)$ (0.0 MeV) in the $^{18}O$ nucleus**

Experimental values are indicated by the filled circles (Kuchta, 1988). The dotted and solid curves indicate the calculations of harmonic-oscillator potential without and with effective charge, respectively. The dashed curves indicate the calculations of Skyrme potential with effective charge. The "+" symbols indicate the calculations of Tassie model.

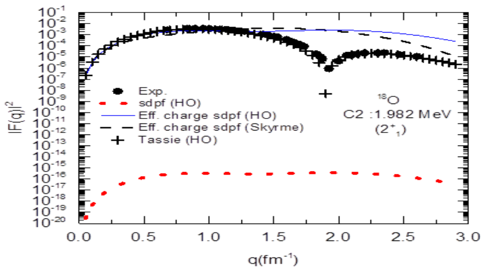


**Figure.4. The longitudinal C2 form factors for the $(2_1^+\ 1)$ (1.982 MeV) in the $^{18}O$ nucleus**

Experimental values are indicated by the filled circles (Kuchta, 1988).

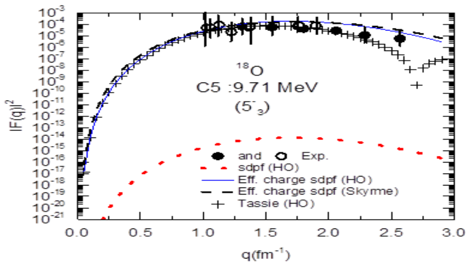


**Figure.5. The longitudinal C5 form factors for the $(5_3^-\ 1)$ (9.971 MeV) in the $^{18}O$ nucleus**

Experimental values are indicated by the open and filled circles.

Fig.6, Shows the longitudinal C0 ($0_1^+$ 0) elastic electron scattering form factor as a function of momentum transfer q in the $^{20}$Ne nucleus. The calculations of model space with HO and Skyrme potentials without effective charge give a good agreement with the experimental data (Knight, 1981).

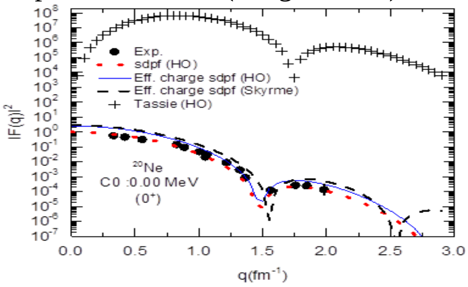


**Figure.6. The longitudinal C0 form factors for the ground state ($0_1^+$ 0) in $^{20}$Ne nucleus**

Experimental values are indicated by the filled circles [Knight, 1981].

Fig.7 and 8 are shown the calculated Charge Density Distributions (CDD) of the $^{18}$O and $^{20}$Ne nuclei which calculated with HO, Skyrme and WS potential in sdpf model space and with TM. The Skyrme potential (dashed curve) calculations give agreement with the experimental data (De Vries, 1987). Table.1, shows the energy levels and reduced transition probabilities B (WL), the results of energy levels gives agreement comparing with experimental data. The B (WL) values give acceptable agreement for available experimental data.

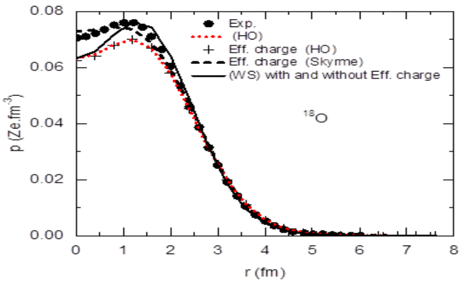


**Figure.7. The dependence of the ground state two body charge density distribution (in fm$^{-3}$) on radius (in fm) for the $^{18}$O nuclei**

Experimental values are indicated by the filled circles (De Vries, 1987)

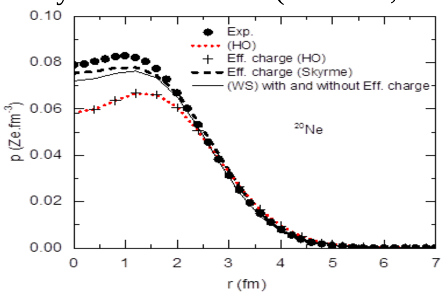


**Figure.8. The dependence of the ground state two body charge density distribution (in fm$^{-3}$) on radius (in fm) for the $^{20}$Ne nuclei**

Experimental values are indicated by the filled circles (De Vries, 1987).

Table (1): Excitation energies and the reduced transition probabilities B (WL) for sdpf-model spaces. The unit of B (EL) is $e^2fm^{2L}$. Experimental excitation energies of Ref. (Tilley, 1998) for $^{17}$O and Ref. (Tilley, 1995) for the $^{18}$O nucleus and Ref. (Tilley, 1998) for the $^{20}$Ne nucleus.

| **nuclei** | **J $^{\pi}$** | **WL** | **E$_x$(MeV±KeV)** | | **B (WL)** | | | | **B(WL) Exp.** |
|---|---|---|---|---|---|---|---|---|---|
| | | | Cal. | Exp. | HO | (HO) $e_{n,p}$=0.35 | (HO) $e_p$=0.35 $e_n$=0.65 | Skyrme $e_{,p}$=0.35 $e_n$=0.65 | |
| $^{17}$O | 1 $/2_1^+$ | C2 | 0.784 | 0.87073 ± 0.10 | 0.55x10$^{-14}$ | 0.941 | 3.248 | 4.04 | 2.18 ±0.16 (Brown, 1980) |
| $^{17}$O | 3 $/2_1^+$ | C2 | 5.594 | 5.0848 ± 0.9 | 1.938x10$^{-13}$ | 0.3296 | 1.137 | 1.482 | 2.05 ± 0.2 (Tilley, 1995) |
| $^{18}$O | $2_1^+$ | C2 | 2.371 | 1.98207± 0.09 | 0.51x10$^{-11}$ | 11.71 | 40.40 | 45.16 | 44.8 ±1.3 (Norum, 1982) |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $^{18}$O | $4_1^+$ | C4 | 4.061 | 3.55484 ± 0.40 | 0.2505x$10^{-5}$ | 1376 | 4744 | 5932 | (9.04±0.9) ×$10^2$ (Tilley, 1995) |
| $^{18}$O | $3_1^+$ | | 6.052 | 5.3778 ± 1.2 | | | | | |
| $^{18}$O | $1_1^+$ | | 11.154 | 8.817 ± 12 | | | | | |
| $^{18}$O | $0_2^+$ | | 4.591 | 3.63376 ± 0.11 | | | | | |
| $^{18}$O | $2_2^+$ | C2 | 4.727 | 3.92044± 0.14 | 0.1180x$10^{-7}$ | 0.4094 | 1.412 | 2.794 | 22.2 ± 1 (Tilley, 1998) |
| $^{18}$O | $3_2^+$ | | | 7.977±4 | | | | | |
| $^{18}$O | $5_3^-$ | C5 | 75.362 | 9.713±7 | 0.56 x$10^{-5}$ | 3846x$10^5$ | 132x$10^6$ | 6.04 x$10^5$ | 3.15x$10^4$ |
| $^{20}$Ne | $2_1^+$ | C2 | 2.042 | 1.63367± 0.015 | 93.59 | 270.5 | 374.4 | 388.7 | 322.9 ±1.8 |
| $^{20}$Ne | $4_1^+$ | | 4.642 | 4.2477 ± 1.1 | | | | | |
| $^{20}$Ne | $0_2^+$ | | 7.257 | 6.725 ± 5 | | | | | |
| $^{20}$Ne | $2_2^+$ | | 7.934 | 7.4219 ± 1.2 | | | | | |
| $^{20}$Ne | $4_2^+$ | | 10.754 | 9.031±7 | | | | | |
| $^{20}$Ne | $2_3^+$ | | 10.835 | 7.8334 ± 1.5 | | | | | |
| $^{20}$Ne | $3_1^+$ | | 10.960 | 9.873±4 | | | | | |
| $^{20}$Ne | $1_1^+$ | | 12.016 | 9.935 ±12 | | | | | |

**4. CONCLUSIONS**

The Large Scale sdpf-shell models calculations can describe the ground state C0 and M1 form factors without applying an effective charge and the positive-parity energy levels for considered nucleus. The model space calculations can't describe the excited states form factors for nucleus with model space consist of only neutrons such as the $^{17,18}$O nucleus. The effective charges didn't enhance the results of form factors because it inherently differs from the experimental shapes, as well as the effective charges for proton aren't affecting the form factors. The inclusion of higher-excited configurations by means of core-polarization (Tassie model) enhances the form factors, where the calculated form factors consist of two peaks but the effective charge of proton still don't affect which indicates the effective charges applied to the nucleons in model space only. For the reduced transition probabilities and charge density distribution , the calculated results of Skyrme potential are closer to the experimental data then indicate that the enhancement of theoretical calculations must base on Skyrme potential. In addition, we note that the effective charges didn't affect the results of charge density distribution.

**REFERENCES**

Brown B.A, and Wildenthal B.H, Status of the nuclear shell model, Annual Review of Nuclear and Particle Science, 38, 1988, 1.

Brown B.A, Arima A, and McGrory J.B, E2 core-polarization charge for nuclei near $^{16}$O and $^{40}$Ca, Nuclear Physics A, 277, 1977, 1.

Brown B.A, Chung W, and Wildenthal B.H, Electromagnetic multi pole moments of ground states of stable odd-mass nuclei in the sd shell, Phys. Rev. C, 22, 1980, 2.

Brown B.A, Wildenthal B.H, Williamson C.F, Rad F.N, Kowalski S, Crannell H, and O'Brien J.T, Shell-model analysis of high-resolution data for elastic and inelastic electron scattering on $^{19}$F, Phys. Rev. C, 32, 1985, 4.

Brussaard and Glademans P.W.M, Shell-model Application in Nuclear Spectroscopy, Amsterdam: North-Holland Publishing Company, 1977.

By R. R. Roy and Nigam B.P, Nuclear physics: theory and experiment, John Wiley and Sons, Inc. New York, 1967.

De Vries H, De Jager C.W, and De Vries C, Nuclear charge-density-distribution parameters from elastic electron scattering, Atomic data and nuclear data tables, 36, 1987, 495.

Donnelly T.W, and Sick I, Elastic magnetic electron scattering from nuclei, Reviews of Modern Physics, 56, 1984, 3.

Jassim K.S, Al-Sammarrae A.A, Sharrad F.I, and Kassim H.A, Elastic and inelastic electron-nucleus scattering form factors of some light nuclei: $^{23}$Na, $^{25}$Mg, $^{27}$Al, and $^{41}$Ca, Phy. Rev.C89, 1, 2014, 014304.

Jassim K.S, and Abdul-Hamza Z.M, longitudinal (c2) form factors with core-polarization effects of some fp-shell nuclei using Tassie model, Armenian journal of physics, 7, 4, 2014, 166.

Jassim K.S, and Kassim H.A, Core polarization effects of some odd sd-shell nuclei using m3y effective nucleon-nucleon interaction, Romanian Journal of Physics, 58, 2013, 3.

Jassim K.S, Nuclear Structure of $^{104,106,108}Sn$ Isotopes Using the NuShell Computer Code, Chinese Journal of Physics, 51, 3, 2013, 441.

Jassim K.S, Radhi R.A, and Hussain N.M, Inelastic magnetic electron scattering form factors of the $^{26}Mg$ nucleus, Pramana Journal of Physics, 86, 1, 2016, 87.

Jassim K.S, Rawaa R.A, Abdul-Nabe A, Core-Polarization Effects on the Isoscalar and Isovector Transitions in $^{24}Mg$ Using Extended Shell Model, Journal of Advanced Physics, 5, 4, 2016, 349.

Jassim K.S, The electron scattering form factor of $^{10}B$, $^{32}S$ and $^{48}Ca$ nuclei, Physica Scripta, 86 (3), 2012, 035202.

Knight E.A, Singhal R.P, Arthur R.G, and Macauley, Elastic scattering of electrons from $^{20,\,22}Ne$, Journal of Physics G: Nucl. Phys., 7, 1981, 8.

Kuchta R, Microscopic boson description of proton-neutron systems: Application to elastic and inelastic electron scattering from O 18 and Ne 20, Physical Review C, 38 (3), 1988, 1460.

Majeed FA, Najim LA, Contribution of high energy configurations to longitudinal and transverse form factors in p-and sd-shell nuclei, Indian Journal of Physics, 89 (6), 2015, 611.

Manley D.M, Berman B.L, Bertozzi W, Buti T.N, Finn J.M, Hersman F.W, Hyde-Wright C.E, Hynes M.V, Kelly J.J, Kovash M.A, and Kowalski S, High-resolution inelastic electron scattering from $^{17}O$, Phys. Rev., C36, 1987, 5.

Mott N.F, The scattering of fast electrons by atomic nuclei, Proceedings of the Royal Society of London, Series A, Containing Papers of a Mathematical and Physical Character, 124, 1929, 794.

Norum B.E, Hynes M.V, Miska H, Bertozzi W, Kelly J, Kowalski S, Rad F.N, Sargent C.P, Sasanuma T, Turchinetz W, and Berman B.L, Inelastic electron scattering from $^{18}O$, Phys. Rev. C, 25, 1982, 4.

Radhi R.A, and Bouchebak A, Microscopic calculations of C2 and C4 form factors in sd-shell nuclei, Nuclear Physics A, 2003, 716.

Radhi R.A, Calculations of elastic and inelastic electron scattering in light nuclei with shell-model wave functions, Michigan State Univ., East Lansing, USA, 1983.

Tassie L.J, A Model of Nuclear Shape Oscillations for Transitions and Electron Excitation, Australian Journal of Physics, 9, 1956, 4.

Tilley D.R, Cheves C.M, Kelley J.H, Raman S, and Weller H.R, Energy levels of light nuclei, A= 20, Nuclear Physics A, 636 (3), 1998.

Tilley D.R, Weller H.R, and Cheves C.M, Energy Levels of Light Nuclei A= 16-17, Nucl. Phys, A564, 1993, 1.

Tilley D.R, Weller H.R, Cheves C.M, and Chasteler R.M, Energy Levels of Light Nuclei A= 18, Nucl. Phys, 595, 1995, 1.

Wong S.S, Introductory nuclear physics, Wiley-VCH Verlag GmbH & Co. KGaA, Germany, 2008.