Thermal radiation effects on Parabolic started Isothermal vertical plate with variable temperature and uniform mass diffusion

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ABSTRACT

This paper deals with the effect of thermal radiation effects on unsteady flow past a parabolic starting motion of the infinite isothermal vertical plate with variable temperature and uniform mass diffusion. The plate temperature is raised linearly with time and the concentration level near the plate is raised uniformly. The Laplace transform technique is used to obtain the expression for velocity, temperature distribution and concentration field of the fluid. The influence of the variable parameter occurring in the problem on velocity, temperature and concentration field is extensively discussed with the help of the graph and table. Radiation-processing have applications in hydrogels, sterilization, natural product enhancement, plastics recycling, ceramic precursors, electronic property materials, ion-track membranes.

KEY WORDS: Parabolic, radiation, isothermal, vertical plate, heat and mass transfer.

1. INTRODUCTION

In modern technology and industrial application non-Newtonian fluid plays an important role hence its study becomes important and interesting.Increasing emergence of non-Newtonian fluids as molten plastics,pulps, emulations and raw materials in a great variety of industrial applications has stimulated a considerable amount of interest in the study of the behaviour of such fluids in motion. It has its practical application in Bio-engineerinh also blood circulation in the artery can be well explained by suitable visco-elastic fluid models of small elasticity. Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

Stokes (1851) presented an exact solution to the Navier-Stokes problem which is the flow of a viscous imcompressible fluid past an impulsively started infinite horizontal plate in its own plane. Soundalgekar (1977) was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite vertical plate by Laplace transform techniques Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant and Ravindra (1990). Agrawal (1999) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal (1998) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique. The objective of this paper is to study the thermal radiation effects on parabolic started infinite vertical plate with variable temperature and uniform mass diffusion. The governing equation are tackled using Laplace-transform techniques. The solutions are in terms of exponential and complementary error function. Such a study will be found useful in chemical, aerospace and other engineering application. In Section 2, mathematical analysis is presented and in section 3, the conclusions are set out.

2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with variable temperature and uniform mass diffusion, in the presence of thermal radiation has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium.The viscous dissipation is assumed to be negligible. The x-axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0 .t'^2$ in its own plane against gravitational field and the plate temperature is made to increase linearly with respect to time, the concentration level near the plate are also raised to C'_w causing convection currents to flow near the plate and the plate is started moving upwards due to impulsive motion gaining velocity $u = u_0 .t'^2$ in its own plane. The plate is infinite in length all the terms in the governing equations will be independent of x and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$\begin{array}{llll} u = 0, & T = T_\infty & C' = C'_\infty & \text{for all } y, t' \le 0 \\ t' > 0: u = u_0 .t'^2, & T = T_\infty + (T_w - T_\infty)At', & C' = C'_w & \text{at } y = 0 \\ u \to 0 & T = T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \quad (4)$$

where $A = \left(\frac{u_0{}^2}{\nu}\right)^{\frac{1}{3}}$.

Here u' is the velocity of the fluid in the x direction, t' the time, g the acceleration due to gravity, β the coefficient of volume expansion, β* the coefficient of thermal expansion with concentration, C' the species concentration in the fluid near the plate, C'_∞ the species concentration in the fluid far away from the plate, ν the kinematic viscosity, T_w the temperature of the plate, ρ the density of the fluid, C_p the specific heat capacity at constant pressure, K the thermal conductivity of the fluid, D the molecular diffusivity, T' is the temperature of the fluid near the plate. The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma(T_\infty^4 - T^4) \quad (5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3 (T_\infty - T) \quad (7)$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0{}^2}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T_w - T_\infty)}{(\nu.u_0)^{1/3}}, \quad Gc = \frac{g\beta^*(C'_w - C'_\infty)}{(\nu.u_0)^{1/3}}, \quad R = \frac{16a^*\sigma.T_\infty{}^3}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}}, \quad \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D} \quad (8)$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (11)$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{lllll} & U=0, & \theta=0, & C=0 & \text{for all } Y,t\le 0 \\ t>0: & U=t^2, & \theta=t, & C=1 & \text{at } Y=0 \\ & U\to 0, & \theta\to 0, & C\to 0 & \text{as } Y\to\infty \end{array} \quad (12)$$

All the physical variables are defined in Nomenclature. These coupled linear equation are then solved by the usual procedure.

2.1. Solution procedure: The dimensionless governing equations (9) to (11) and the corresponding initial and boundary conditions (12) are tackled using Laplace transform technique.

$$U(\mathrm{y,t}) = \frac{t^3}{3}\left[\left(3+12\eta^2+4\eta^4\right)erfc(\eta) - \eta\left(10+4\eta^2\right)\frac{e^{-\eta^2}}{\sqrt{\pi}}\right]$$

$$+\left(\frac{Gr}{b(1-\mathrm{pr})}\right)erfc(\eta)$$

$$-\left(\frac{Gr}{2b(1-\mathrm{pr})}\right)\left\{e^{bt}\left[e^{2\eta\sqrt{bt}}erfc(\eta+\sqrt{bt}) + e^{-2\eta\sqrt{bt}}erfc(\eta-\sqrt{bt})\right]\right\}$$

$$-\left(\frac{Gc\,t}{(1-sc)}\right)\left[(1+2\eta^2)\,\mathrm{erfc}(\eta) - 2\eta\frac{e^{-\eta^2}}{\sqrt{\pi}}\right]$$

$$+e^{-2\eta\sqrt{atpr}}erfc(\eta\sqrt{pr}-\sqrt{at})$$

$$-\frac{pr\eta\sqrt{t}}{2\sqrt{R}}\left\{e^{-2\eta\sqrt{atpr}}erfc(\eta\sqrt{pr}-\sqrt{at}) - e^{2\eta\sqrt{atpr}}erfc(\eta\sqrt{pr}+\sqrt{at})\right\}$$

$$+\left(\frac{Gr}{b^2(1-\mathrm{pr})}\right)\left\{\frac{e^{bt}}{2}\left[e^{2\eta\sqrt{pr(b+a)t}}erfc(\eta\sqrt{pr}+\sqrt{(\mathrm{b+a})\,\mathrm{t}} + e^{-2\eta\sqrt{pr(b+a)t}}erfc(\eta\sqrt{pr}-\sqrt{(\mathrm{b+a})\,\mathrm{t}}\right]\right\}$$

$$+\left(\frac{Gr}{b(1-\mathrm{pr})}\right)\left[\left(1+2\eta^2\right)erfc(\eta) - 2\eta\frac{e^{-\eta^2}}{\sqrt{\pi}}\right] \quad (13)$$

$$\theta(y,t) = \frac{t}{2}\{e^{2\eta\sqrt{prat}}erfc(\eta\sqrt{pr}+\sqrt{at}) + e^{-2\eta\sqrt{prat}}erfc(\eta\sqrt{pr}-\sqrt{at})\}$$
$$-\frac{pr\eta\sqrt{t}}{2\sqrt{R}}\{e^{-2\eta\sqrt{prat}}erfc(\eta\sqrt{pr}-\sqrt{at}) - e^{2\eta\sqrt{prat}}erfc(\eta\sqrt{pr}+\sqrt{at})\} \quad (14)$$

$$C(y,t) = erfc(\eta\sqrt{sc}) \quad (15)$$

where, $a=\dfrac{R}{Pr}$, $b=\dfrac{R}{1-Pr}$, $c=\dfrac{Gr}{2b(1-Pr)}$, $d=\dfrac{Gct^2}{6(1-Sc)}$ and $\eta=\dfrac{Y}{2\sqrt{t}}$ is the similarity parameter.

Nomenclature: C'-species concentration in the fluid, C - dimensionless concentration, C_p - specific heat at constant pressure, D - mass diffusion coefficient, Gc - mass Grashof number, Gr - thermal Grashof number, g - acceleration due to gravity, K - thermal conductivity, Pr - Prandtl number, Sc - Schmidt number, T - temperature of the fluid near the plate, t' - time, u - velocity of the fluid in the x-direction, u_0 - velocity of the plate, u - dimensionless velocity, y - coordinate axis normal to the plate, Y - dimensionless coordinate axis normal to the plate.

Greek symbols: β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of expansion with concentration, μ - coefficient of viscosity, ν - kinematic viscosity, ρ - density of the fluid, τ - dimensionless skin-friction, θ - dimensionless temperature, η - similarity parameter, $erfc$ - complementary error function.

Subscripts: w - conditions at the wall, ∞ - free stream conditions.

3. RESULTS AND DISCUSSION

In order toget the physical insight into the problem, we have evaluated the numerical values of U,θ,C for diffferent values of Pr(= 0.71, 7.0)Sc, Gr Gc and R.The numerical values of the Schmidt number Sc are chosen such that they represent a reality in the case of air and they are presented in Table:1. These values of Schmidt number are chosen to represent various species at low concentration in air at approximately 25˚C and 1 atmosphere. The numerical values of the concentration profiles are evaluated from expression (15) and this plotted on Fig (1) for different values of the Schmidt number.We observe that this figure shows an increase in the Schmidt number leads to a decrease in the concentration profiles.

The temperature profiles are plotted in Fig(2) for different values of thermal radiation parameter R with t=0.2. It is observed that the temperature increases with decreasing radiation parameter. The temperature profiles are derived from (14) and these are shown on Fig (3) for air (Pr=0.71) and water (Pr=7.0) with t=0.2. We observe that there is a fall in temperature with increasing Prandlt number. The temeprature profiles increases with increase in time which is shown in fig(4).

Table.1. Schmidt number values of Species

Species	Schmidt number	Name
H_2	0.16	Hydrogen
He	0.3	Helium
H_2O	0.6	Water Vapour
CO_2	1.00	Carbon dioxide
NH_3	0.78	Ammonia
$C_6H_5CH_2CH_3$	2.01	Ethyl Benzene

The numerical values of the velocity profiles are evaluated from expression(13) and this is plotted on Fig(5) for different values of Sc for air.It is seen that an increase in the Schmidt number leads to a fall in the velocity. InFig(6) the velocity profiles are shown for different values of mass Grashof number and thermal Grashof number and we conclude that there is a rise in velocity when Gr and Gc increase.

The effect of velocity for different values of the radiation parameter ($R = 2,5,10), Gr = 5 = Gc$ and $t = 0.8$ are shown in figure(7). The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

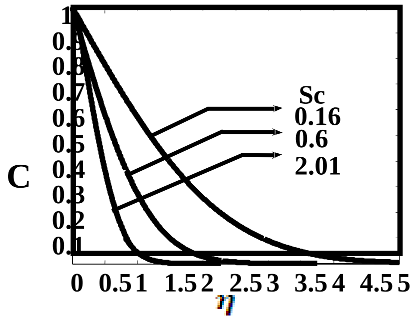

Figure 1:Concentration Profiles for different values of Sc

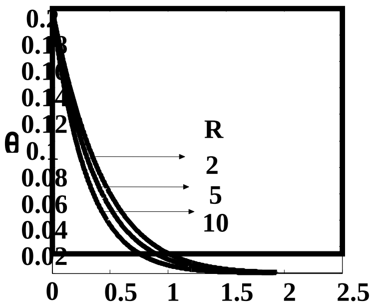

Figure 2:Temperature Profiles for different values of R

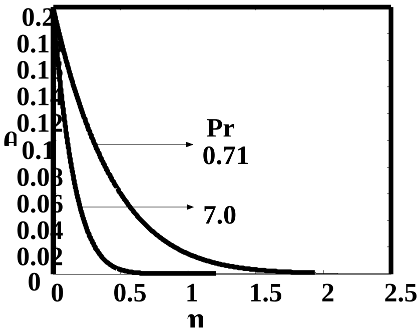

Figure 3:Temperature Profiles for different values of Pr

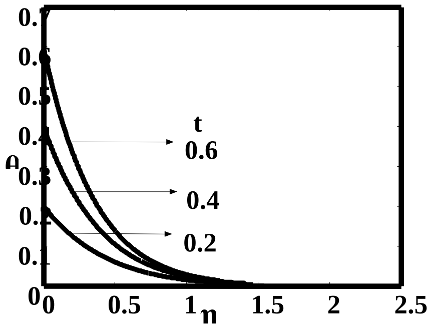

Figure 4:Temperature Profiles for different values of t

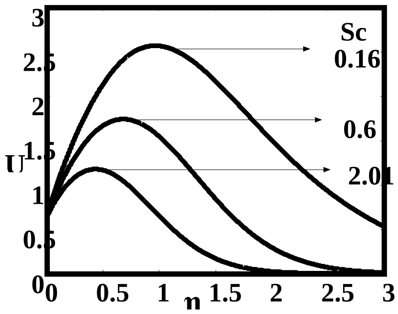

Figure 5:Velocity Profiles for different values of Sc

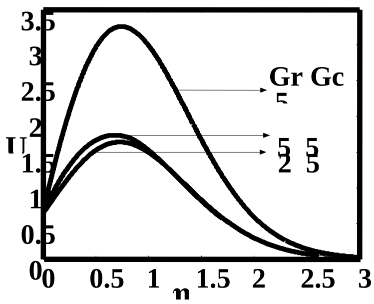

Figure 6:Velocity Profiles for different values of Gr and Gc

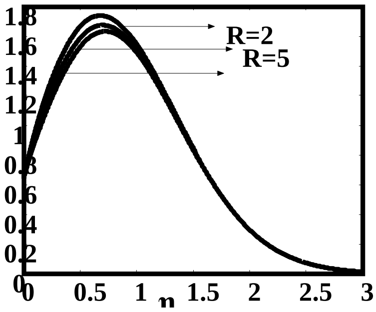

Figure 7:Velocity Profiles for different values of R

4. CONCLUSION

The theoretical solution of flow past a parabolic starting motion of the infinite isothermal vertical plate with variable temperature and uniform mass diffusion, in the presence thermal radiation has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal radiation parameter, thermal Grashof number d mass Grash of number Schmidt number and Prandlt number are studied graphically. The conclusions of the study are as follows:

a) The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter.

b) The temperature of the plate increases with decreasing values of the thermal radiation parameter and there is a fall in temperature with increasing Prandlt number.The plate temeprature increases with increase in time.

c) The plate concentration increases with decreasing values of the Schmidt number.

REFERENCES

Agrawal A.K, and Samria N.K, and Gupta S.N, free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic field, Journal of Heat and mass transfer, 20, 1998, 35-43.

Agarwal A.K, Samria N.K, and Gupta S.N, Study of heat and mass transfer past a parabolic started infinite vertical plate, Journal of, Heat and mass transfer, 21, 1999, 67-75.

Basanth Kumar Jha, and Ravindra Prasad, Free convection and mass transfer effects on the flow past an accelerated vertical plate with heat sources, Mechanics Research Communications, 17, 1990, 143-148.

Gupta A.S, Pop I, Soundalgekar V.M, Convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluis, Rev. Roum. Sci. Techn. Mec. Apl., 24, 1979, 561-568.

Hossain M.A, and Shayo L.K, The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, 125, 1986, 315-324.

Kafousias N.G, and Raptis A.A, Mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection, Rev. Roum. Sci. Techn.Mec. Apl., 26, 1981, 11-22.

Singh A.K, and Naveen Kumar, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space Science, 98, 1984, 245-258.

Singh A.K, and Singh J, Mass transfer effects on the flow past an accelerated vertical plate with constant heat flux, Astrophysics and Space Science, 97, 1983, 57-61.

Soundalgekar V.M, Effects of mass transfer on flow past a uniformly accelerated vertical plate, Letters in heat and mass transfer, 9, 1982, 65-72.

Soundalgekar V.M, J. Heat Tr. (Tr. ASME) 99C, 1977, 499.

Stokes G.G, Camb.Phil.Trans.IX, 1851, 8.