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New Classes of α-valuation and total product cordial graph

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Abstract

In 1966 Rosa defined an $\alpha-$labeling (or $\alpha-$valuation) as a graceful labeling with the property that there exists an integer λ so that for each edge xy either $f(x)\le\lambda< f(y)$ or $f(y)\le\lambda< f(x)$. It follows that such a λ must be the smaller of the two nodes labels that yield the edge labeled 1. A simple and finite graph G(V,E) is said to be graceful if there exists an injection $f:V(G)\to\{0,1,2,\dots,q\}$ such that $f':E(G)\to\{1,2,\dots,q\}$ defined by $f'(uv)=|f(u)-f(v)|, uv\in E(G)$ is a bijection. The graph QS_n is called Quadrilateral Snake graph. It is defined as series connection of non-adjacent vertices of 'n' number of cycle C_4 and these vertex set V and edge set E have described below

$$V(QS_n)=\{c_k\}_{k=1}^{n+1}\cup\{u_i\}_{i=1}^{n}\cup\{v_j\}_{j=1}^{n}$$

$$E(QS_n)=\{c_k u_k\}_{k=1}^{n}\cup\{c_k v_k\}_{k=1}^{n}\cup\{u_k c_{k+1}\}_{k=1}^{n}\cup\{v_k c_{k+1}\}_{k=1}^{n}$$

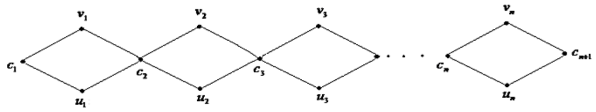

Figure.1. Quadrilateral Snake Graph QS_n

A total product cordial labeling of a graph G is a function $f:(V(G)\cup E(G))\to\{0,1\}$ such that $f(xy)=f(x)f(y)$ where x, y $\in$ V(G), xy $\in$ E(G) and the total number of 0 and 1 are balanced. That is, if $v_f(i)$ and $e_f(i)$ denote the set of nodes and edges which are labeled as i for $i=0,1$ respectively, then, $|(v_f(0)+e_f(0))-(v_f(1)+e_f(1))|\le 1$.

we prove that (i) The graph $G=P_m(QS_n)$ *for every* $m\ge 2$ *and* $n\ge 1$ admits an $\alpha-$valuation.

$G=P_m(QS_n)^t$, $m\equiv 0(\mathrm{mod}\,2), m\ge 2, \forall n\ge 1, t\ge 1$ is total product cordial graph.

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KEY WORDS : Graceful labeling, $\alpha-$valuation, Path, Binary labeling, Cordial labeling, Total product cordial labeling and Quadrilateral Snake graph.

1. INTRODUCTION

Cahit (1987) introduced cordial labeling which is otherwise termed as binary vertex labeling. A binary vertex labeling of a graph G is called a cordial labeling if $|(v_f(0)-v_f(1)|\le 1$ and $|e_f(0))-e_f(1))|\le 1$.

A binary nodes labeling of graph G with induced edge labeling $f^*:E(G)\to\{0,1\}$ defined by $f^*(e=uv)=f(u)f(v)$ is called a product cordial labeling if $|(v_f(0)-v_f(1)|\le 1$ and $|e_f(0))-e_f(1))|\le 1$.

Sethuraman and Dhavamani (2000) have studied shell graphs. Sethuraman and Selvaraju (2001), have studied one vertex union of non-isomorphic complete bipartite graphs. For these topics one may refer to the excellent survey paper of Gallian (2013) (Gallian, 2011)

The graph $G=P_m(QS_n)^t$ is defined as an isomorphic Quadrilateral snake't' copies gluing with each nodes of path P_m, n is the number of Blocks (i.e., C_4) of Quadrilateral snake QS_n in one copy.

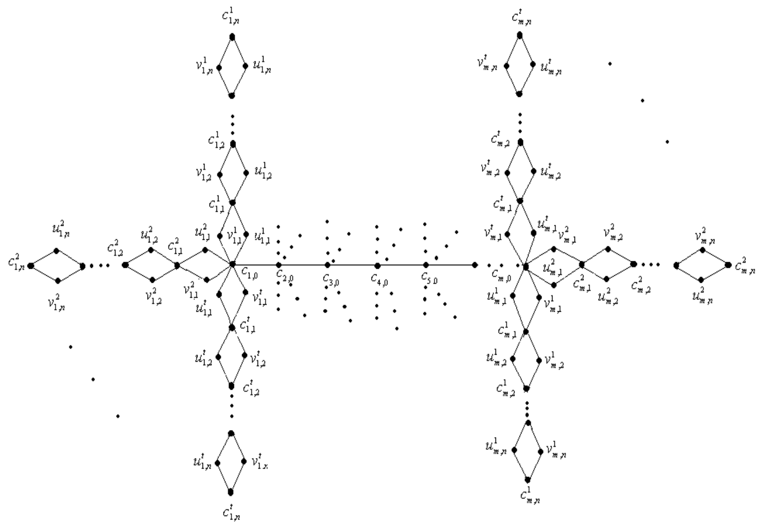

Figure.2. The graph $G = P_m(QS_n)^t$

Theorem – 1.1

The graph $G = P_m(QS_n)$, $\forall\ m \geq 2$, $n \geq 1$ admits an α – valuation.

Proof: The graph $G = P_m(QS_n)$ has $m(3n+1)$ vertices and $m(4n+1)-1$ edges.

The graph of $G = P_m(QS_n)$ is given in Figure 3

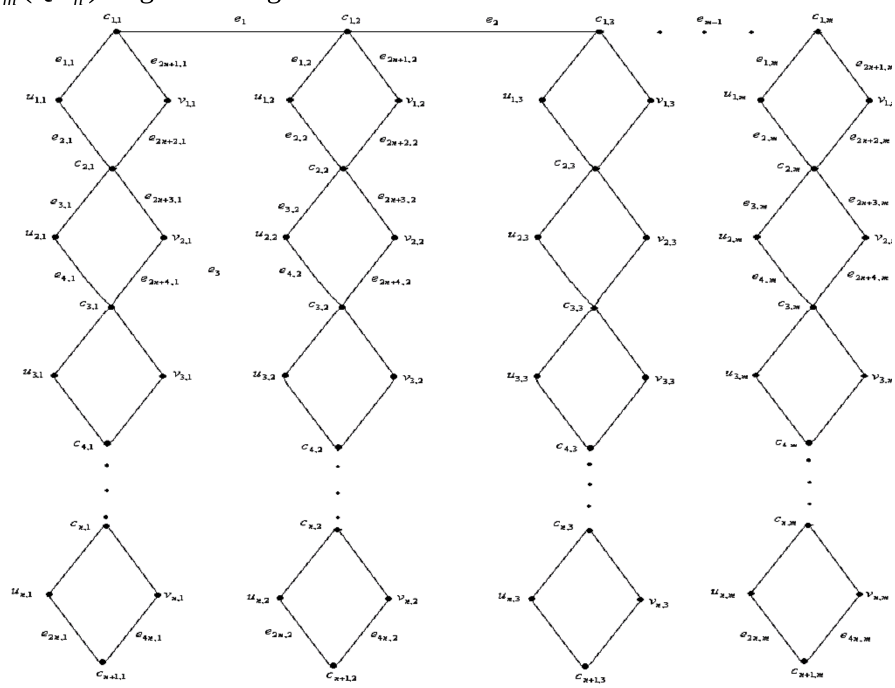

Figure.3. The graph $P_m(QS_n)$

The graceful labeling for vertices of G is conveniently defined as set A, B and C as follows

$$A = \left\{ f(c_{i,j}) \,/\, \begin{array}{ll} (i-1) + \dfrac{(j-1)}{2}(4n+1), & i = 1,2,3,\ldots,n+1, \quad j = 1,3,5,\ldots,m, \ m \text{ is odd} \\ (2m - j + 2)\dfrac{(4n+1)}{2} - 4n + i - 2, & i = 1,2,3,\ldots,n+1, \quad j = 2,4,\ldots,m, \ m \text{ is even} \end{array} \right\}$$

$$B = \left\{ f\left(u_{i,j}\right) / \begin{array}{ll} m(4n+1)-(j-1)(2n+\frac{1}{2})-i, & i=1,2,3,...,n, \quad j=1,3,5,...,m, \ m \ is \ odd \\ 2n+(4n+1)\frac{(j-2)}{2}-(i-1), & i=1,2,3,...,n, \quad j=2,4,6,...,m, \ m \ is \ even \end{array} \right\}$$

$$C = \left\{ f\left(v_{i,j}\right) / \begin{array}{ll} m(4n+1)-\frac{(j-1)}{2}(4n+1)-2n-i, & i=1,2,3,...,n, \ j=1,3,5,...,m, \ m \ is \ odd \\ 4n+(4n+1)\frac{(j-2)}{2}-(i-1), & i=1,2,3,...,n, \ j=2,4,6,...,m, \ m \ is \ even \end{array} \right\}$$

From the above vertex labeling, observe that, set A form a monotonically increasing sequence for each j and the set B&C form a monotonically decreasing sequence for each j. $A \cup B \cup C = \{ 0,1,2,...,m(3n+1) \}$ and

$A \cap B \cap C = \phi$

Therefore, the set A, B&C are distinct.

Edge labeling are defined by

$A' = \left\{ f'\left(e_{i,j}\right) / \ (4n+1)(m-j+1)-i, \quad 1 \le i \le 4n, \quad j \ is \ odd, \quad 1 \le j \le m \right\}$

$B' = \left\{ f'\left(e_{i,j}\right) / \ (4n+1)(m-j+2)-6n+i-2, \quad 1 \le i \le 2n, \quad j \ is \ even, \quad 2 \le j \le m \right\}$

$C' = \left\{ f'\left(e_{i,j}\right) / \ (4n+1)(m-j+2)-10n+i-2, \quad 2n < i \le 4n, \quad j \ is \ even, \quad 2 \le j \le m \right\}$

$D' = \left\{ f'\left(e_k\right) / \ (4n+1)(m-k+1)-4n-1, \quad 1 \le k \le m-1 \right\}$

Observe that the values in the sets $A', B', C' \ and \ D'$ are all distinct and $A' \cup B' \cup C' \cup D' = \{ 1,2,3,...,m(4n+1)-1 \}$

$A' \cap B' \cap C' \cap D' = \phi$

From the above assignment the labeling of nodes and edges are distinct.

Hence the graph $G = P_m(QS_n)$ is a graceful graph.

In $\lambda = \frac{m(4n+1)-2}{2}$ or $\lambda = \frac{m(4n+1)-3}{2}$ the value of m is either even or odd, accordingly.

It is clear that $f(u) \le \lambda < f(v)$ for every edge uv of $P_m(QS_n)$

Thus, $P_m(QS_n)$ admits a $\alpha-$valuation.

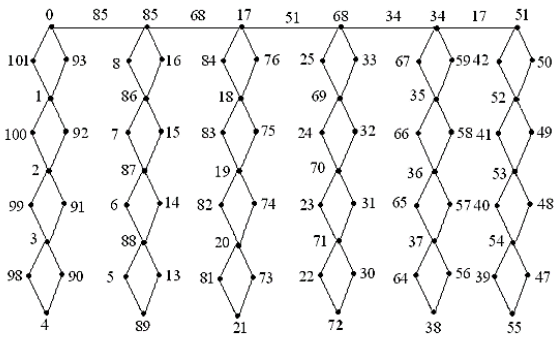

Figure.4. $\alpha-$**valuation of** $P_6(QS_4)$ **with** $\lambda = 50$

Theorem -1.2

The graph $G = P_m(QS_n)^t$, $m \equiv 0(\bmod 2), m \ge 2, \forall n \ge 1, t \ge 1$ is total product cordial graph.

Proof: The graph $G = P_m(QS_n)^t$ has $m(1+3tn)$ vertices and $m(4tn+1)-1$ edges.

The graph of $G = P_m(QS_n)^t$ is given in figure 2

Vertices of $G = P_m(QS_n)^t$ are arranged as a sequence in certain order and then matched term wise with a particular sequence of 0 and 1

Arrange the nodes of path P_m as a sequence, A= $\{ c_{1,0}, c_{2,0}, c_{3,0}, c_{4,0}, \ldots, c_{m,0} \}$

The edges of path P_m as a sequence $B = \{ c_{1,0}c_{2,0}, c_{2,0}c_{3,0}\ c_{3,0}c_{4,0}, \ldots, c_{m-1,0}c_{m,0} \}$

The edges of Quadrilateral snake as a sequence $C = \{ c_{1,0}u_{1,1}^1, u_{1,1}^1 c_{1,1}^1\ c_{1,1}^1 u_{1,2}^1, \ldots, u_{1,n}^1 c_{1,n}^1 \}$

$(QS_n)^t$ is incident with the nodes '0'

The edges of Quadrilateral snake as a sequence $D = \{ c_{1,0}v_{1,1}^1, v_{1,1}^1 c_{1,1}^1\ c_{1,1}^1 v_{1,2}^1, \ldots, v_{1,n}^1 c_{1,n}^1 \}$

$(QS_n)^t$ is incident with the nodes '0'

The edges of Quadrilateral snake as a sequence $E = \{ c_{1,0}u_{1,1}^1, u_{1,1}^1 c_{1,1}^1\ c_{1,1}^1 u_{1,2}^1, \ldots, u_{1,n}^1 c_{1,n}^1 \}, (QS_n)^t$

is incident with the nodes assigned the label '1'

Arrange the edges of Quadrilateral snake as a sequence

$F = \{ c_{1,0}v_{1,1}^1, v_{1,1}^1 c_{1,1}^1\ c_{1,1}^1 v_{1,2}^1, \ldots, v_{1,n}^1 c_{1,n}^1 \}$, $(QS_n)^t$ is incident with the vertices assigned the label '1

Table.1. Vertex labeling of $G = P_m(QS_n)^t$

Nature of the vertices of G	The 0-1sequence for term wise matching with the sequence of vertices of G
A	$(0)^{\frac{m}{2}}(1)^{\frac{m}{2}}$
$u_{i,j}^t, v_{i,j}^t\ \forall\ j,t,\ 1 \le i \le \frac{m}{2}$	(0)
$u_{i,j}^t, v_{i,j}^t\ \forall\ j,t,\ \frac{m}{2} < i \le m$	(1)
$c_{i,j}^{\ t}\ \forall\ j,t,\ 1 \le i \le \frac{m}{2}$	(0)
$c_{i,j}^{\ t}\ \forall\ j,t,\ \frac{m}{2} < i \le m$	(1)

The total product cordial labeling of the vertices of $G = P_m(QS_n)^t$ is given in Table 1.

Table.2.Edge labeling of $G = P_m(QS_n)^t$

Nature of the edge labels sequence for the set	**The 0-1sequence for term wise matching with the sequence of edges of G**
B	$(0)^{\frac{m}{2}}(1)^{\frac{m-2}{2}}$
C&D	$(0)^{2n}$
E&F	$(1)^{2n}$

The total product cordial labeling of the edges of $G = P_m(QS_n)^t$ is given in Table.2

Table.3. Relation between vertices and edges of $P_m(QS_n)^t, m \equiv 0\ (\mathrm{mod}\ 2), m \ge 2$

Number of copies (t)	**Number of block (n)**	**Relation between $\lvert V_0 \rvert$ and $\lvert V_1 \rvert$**	**Relation between $\lvert E_0 \rvert$ and $\lvert E_1 \rvert$**	$\lvert (v_f(0)+e_f(0)) - (v_f(1)+e_f(1)) \rvert$
t is even (or) t is odd	n is even (or) n is odd	$\lvert V_0 \rvert = \lvert V_1 \rvert$	$\lvert E_0 \rvert = \lvert E_1 \rvert + 1$	$\lvert (v_f(0)+e_f(0)) - (v_f(1)+e_f(1)) \rvert = 1$

From the Table 3, it is clear that $|V_0| = |V_1|$, $|E_0| = |E_1| + 1$

The graph $G = P_m(QS_n)^t$, $m \equiv 0 (\mathrm{mod}\, 2), m \geq 2, \forall n \geq 1, t \geq 1$ is total product cordial graph.

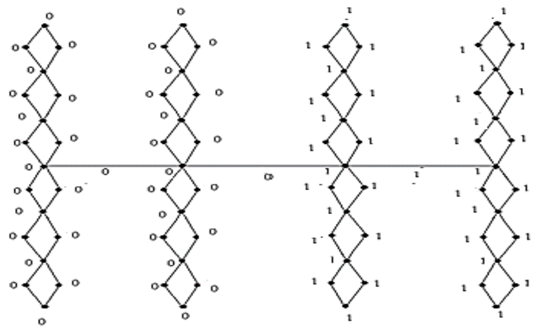

Figure.5.Total product cordial labeling of $P_4(QS_3)^2$

2. CONCLUSION

It has been proved that the graph $G = P_m(QS_n)$ *for every* $m \geq 2$ *and* $n \geq 1$ admits an $\alpha-$valuation and the graph $G = P_m(QS_n)^t$, $m \equiv 0 (\mathrm{mod}\, 2), m \geq 2, \forall n \geq 1, t \geq 1$ is total product cordial graph.

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