

# Solution of system of Boolean equation to give the input of the circuit

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## ABSTRACT

This paper presents the Boolean expression for network analysis. A method is proposed to get the equivalent network for the given network using the postulation. Each network can frame several Boolean functions and that is considered as to be system of Boolean equations, with the help of system of Boolean equation with the output, input can be obtained.

**KEY WORDS:** Boolean expression, Boolean Function, System of Boolean equations.

## 1. INTRODUCTION

George Boole introduced Boolean algebra in his first book *The Mathematical Analysis of Logic* (1847), and extended the concept of Boolean algebra in his *An Investigation of the Laws of Thought* (1854). The concept is developed by Sheffer in 1913. Boolean algebra is used in various fields like digital electronics (which is useful for writing the modern programming languages), Set theory and statistics.

Even Boolean algebra is the origin of abstract algebra and mathematical logic; it predated the developments of it. In late 19<sup>th</sup> century by Jevons, Schroder, Huntington, and others are perfected the Boolean algebra as a mathematical structure to reach the modern conception. For example, one can write equation in the algebra of sets by changing them as the equation of Boolean algebra.

In 1930s, Claude Shannon observed used the Boolean algebra rules in switching algebra and design circuits by algebraic means in terms of logic gates. Thus, the difficult switching algebra can be constructed with two elements Boolean algebra. Now a days, Boolean functions are efficiently used to construct the logic circuits.

In 1914, "The Algebra of Logic" written by the French mathematician Louis Couturat explains clearly the difference of Boole's algebraic system and logic. In 2011, Janet Heine Barnett defined the circuits can be expressed as a equation using Boolean function and in 2010 find the solution of system of Boolean equation. In this paper system of equations can be connected to the circuits with output and by solving this system of equations, inputs can be obtained.

**Preliminaries:** In this section we concentrate on Boolean algebra, Circuit design which will be useful in later text.

**Boolean function:** If  $x_1, x_2 \dots x_n$  are Boolean variables,  $f: \{x_1, x_2 \dots, x_n\} \rightarrow \{0, 1\}$  is called Boolean function of degree  $n$ .

**Boolean Expression:** A Boolean expression in  $n$  Boolean variables  $x_1, x_2 \dots x_n$  is a finite string of symbols formed recursively.

**Boolean Expression using Boolean function:**

(i) **One variable:** Assume,

$$f(x) = ax + b(1 - x),$$

take  $x = 1$ , we have

$$f(1) = a.$$

Again, in the same equation making  $x = 0$ , we have

$$f(0) = b.$$

Hence, the values  $a$  and  $b$  are determined, and substituting them in the first equation can be rewritten as:

$$f(x) = f(1)x + f(0)(1 - x);$$

We denote  $x'$  for  $1 - x$ , so that above equation is rewritten as

$$f(x) = f(1)x + f(0)x';$$

(ii) **Two variable:**  $f(x, y) = f(1,1)xy + f(1,0)x(1 - y) + f(0,1)(1 - x)y + f(0,0)(1 - x)(1 - y)$ , (1)

We can use the above equation to verify that the following table of function values defines the function represented by  $f(x, y) = xy + x'y$ .

$x$	$y$	$f(x, y)$
0	0	0
0	1	1
1	0	0
1	1	1

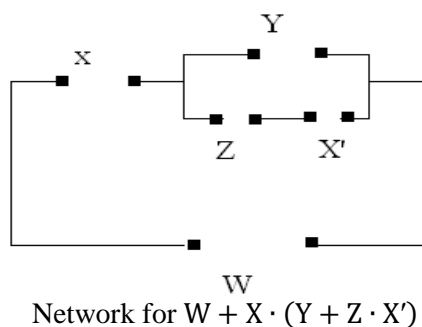
Provided the output does not contain all '0' and all '1'.

**(iii) Three variable:**

$$f(x, y, z) = xyzf(1,1,1) + xy'zf(1,0,1) + xyz'f(1,1,0) + xy'z'f(1,0,0) + x'yzf(0,1,1) + x'y'zf(0,0,1) + x'yz'f(0,1,0) + x'y'z'f(0,0,0)$$

$$\text{where } x' = 1 - x, y' = 1 - y, z' = 1 - z \quad (2)$$

**Circuit Design:** The application of Boolean algebra to circuits provided an actual physical representation for the corresponding symbolic operations. Diagrams depicting the two types of connections and the corresponding operations are shown in Figure 1 below. An example of how to represent a more complicated circuit with an algebraic equation, based on an example from Shannon, is shown in Figure 2

**Figure 1****Figure 2**Network for  $W = X + (Y + Z \cdot X')$ 

**Algebraic Identities:** The above diagrams such as these not only to represent circuits, but also illustrate Boolean algebra identities. The following excerpt gives his description of the basic Boolean identities for circuits.

$$X + Y = Y + X$$

$$XY = YX$$

$$X + (Y + Z) = (X + Y) + Z$$

$$X(YZ) = (XY)Z$$

$$X(Y + Z) = XY + XZ$$

$$X + (YZ) = (X + Y)(X + Z)$$

$$1 \cdot X = X$$

$$0 + X = X$$

$$1 + X = 1$$

$$0 \cdot X = 0$$

$$X + X' = 1$$

$$XX' = 0$$

$$0' = 1$$

$$1' = 0$$

$$(X')' = X$$

$$(X + Y + Z + \dots)' = X' \cdot Y' \cdot Z' \dots$$

$$(X \cdot Y \cdot Z \dots)' = X' + Y' + Z' \dots$$

$$X = X + X = X + X + X = \dots$$

$$X = X \cdot X = X \cdot X \cdot X = \dots$$

$$X + XY = X$$

$$X(X + Y) = X$$

**Equivalent circuits:** Already Shannon's applied Boolean algebra to the study of circuits equivalent in operating characteristics to a given circuit. The hindrance of the given circuit is written down and manipulated according to

the rules. Each different resulting expression represents a new circuit equivalent to the given one. In particular, expressions may be manipulated to eliminate elements which are unnecessary, resulting in simple circuits.

For example, the following computation confirms that the element  $Z$  can be eliminated of the network diagram in Figure 2, a fact which careful examination of the network diagram in figure 2 also reveals:

$$W + X \cdot (Y + Z \cdot X') = W + XY + X(ZX') = W + XY + (XX')Z = W + XY + 0 \cdot Z = W + XY$$

### Solution of System of Boolean Equations to Give the Input of the Circuit (Two variables):

**Redefining function:** Consider a two variable circuit (say  $x, y$ ), Then we can form four types of combination (say  $xy, x'y', xy', x'y$ ) that is  $2^n$  terms (here  $2^2 = 4$  terms).

$$g(x, y) = \varphi(1)xy + \varphi(2)x'y' + \varphi(3)xy' + \varphi(4)x'y \quad (3)$$

Where  $\varphi(1), \varphi(2), \varphi(3)$  and  $\varphi(4)$  takes the values 0 and 1.

Values	$g_1(x, y)$	$g_2(x, y)$
$\varphi(1)$	1	1
$\varphi(2)$	0	1
$\varphi(3)$	0	0
$\varphi(4)$	1	0

$$\therefore g_1(x, y) = 1 \cdot xy + 0 \cdot x'y' + 0 \cdot xy' + 1 \cdot x'y = xy + x'y \quad (4)$$

And  $g_2(x, y) = 1 \cdot xy + 1 \cdot x'y' + 0 \cdot xy' + 0 \cdot x'y = xy + x'y' \quad (5)$

**Solving System of Boolean Equation:** We can use the following table,

$x$	$y$	values	$g_1(x, y)$	$g_2(x, y)$
?	?	$\varphi(1)$	1	1
?	?	$\varphi(2)$	0	1
?	?	$\varphi(3)$	0	0
?	?	$\varphi(4)$	1	0

Assume  $x$  and  $y$  are two unknowns, according to algebraic rule two unknowns need two equations to solve that is from the table, we have,  $g_1(x, y) = x'y + xy$  and  $g_2(x, y) = x'y' + xy$  are two functions with four different outputs.

By using algebraic identities,

The  $g_1(x, y)$  becomes,

$$x'y + xy = (x' + x)y = 1 \cdot y = y$$

and the  $g_2(x, y)$  becomes,

$$x'y' + xy = (1 - x)(1 - y) + xy = 1 - x - y + 2xy$$

(i) From the first row we take the outputs, we have  $g_1(x, y) = 1$  and  $g_2(x, y) = 1$ .

Therefore the equation becomes,

$$y = 1 \text{ and } 1 - x - y + 2xy = 1$$

Solve the above two equation, we get  $x = 1$ .

$\therefore x = 1$  and  $y = 1$  which is the corresponding input

(ii) From the first row we take the outputs, we have  $g_1(x, y) = 0$  and  $g_2(x, y) = 1$ .

Therefore the equation becomes,

$$y = 0 \text{ and } 1 - x - y + 2xy = 1$$

Solve the above two equation, we get  $x = 0$ .

$\therefore x = 0$  and  $y = 0$  which is the corresponding input.

(iii) From the first row we take the outputs, we have  $g_1(x, y) = 0$  and  $g_2(x, y) = 0$ .

Therefore the equation becomes,

$$y = 0 \text{ and } 1 - x - y + 2xy = 0$$

Solve the above two equation, we get  $x = 1$ .

$\therefore x = 1$  and  $y = 0$  which is the corresponding input.

(iv) From the first row we take the outputs, we have  $g_1(x, y) = 1$  and  $g_2(x, y) = 0$ .

Therefore the equation becomes,

$$y = 1 \text{ and } 1 - x - y + 2xy = 0$$

Solve the above two equation, we get  $x = 0$ .

$\therefore x = 0$  and  $y = 1$  which is the corresponding input.

$x$	$y$	values	$g_1(x, y)$	$g_2(x, y)$
1	1	$\varphi(1)$	1	1
0	0	$\varphi(2)$	0	1
1	0	$\varphi(3)$	0	0
0	1	$\varphi(4)$	1	0

**Possible Redefined Equations:**

values	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_9$	$g_{10}$	$g_{11}$	$g_{12}$	$g_{13}$	$g_{14}$
$\varphi(1)$	1	0	1	1	0	0	0	0	0	0	1	1	1	1
$\varphi(2)$	0	1	0	1	0	1	0	0	1	1	0	0	1	1
$\varphi(3)$	1	0	0	0	1	1	0	1	0	1	0	1	0	1
$\varphi(4)$	0	1	1	0	1	0	1	0	0	1	0	1	1	0

**Verification:**

$x$	$y$	$f_1(x, y)$	$f_2(x, y)$
1	1	1	1
0	0	0	1
1	0	0	0
0	1	1	0

$$f_1(x, y) = xy + x'y \text{ and } f_2(x, y) = xy + x'y' \quad (7)$$

The above equation is same as (5) and (6).

**Solution of System of Boolean Equation to Give the Input of the Circuit (Three variables):**

**Redefining function:** Consider a three variable circuit (say  $x, y, z$ ), Then we can form eight types of combination (say  $xyz, x'y'z', xy'z, x'yz, xyz', x'y'z, xy'z', x'yz'$ ) that is  $2^n$  terms (here  $2^3 = 8$  terms).

$$g(x, y, z) = \varphi(1)xyz + \varphi(2)x'y'z' + \varphi(3)x'yz' + \varphi(4)xy'z + \varphi(5)xy'z' + \varphi(6)xyz' + \varphi(7)x'yz + \varphi(8)x'y'z \quad (8)$$

Where  $\varphi(1), \varphi(2), \varphi(3), \varphi(4), \varphi(5), \varphi(6), \varphi(7)$  and  $\varphi(8)$  takes the values 0 and 1.

**Solving System of Boolean Equation:** We can use the following table,

$x$	$Y$	$z$	values	$g_1(x, y, z)$	$g_2(x, y, z)$	$g_3(x, y, z)$
?	?	?	$\varphi(1)$	1	1	1
?	?	?	$\varphi(2)$	0	1	1
?	?	?	$\varphi(3)$	0	0	1
?	?	?	$\varphi(4)$	0	0	0
?	?	?	$\varphi(5)$	0	0	0
?	?	?	$\varphi(6)$	0	0	0
?	?	?	$\varphi(7)$	0	0	0
?	?	?	$\varphi(8)$	0	0	0

Assume  $x, y$  and  $z$  are three unknowns, according to algebraic rule three unknowns need three equations to solve, that is from the table, we have,  $g_1(x, y, z) = xyz$ ,  $g_2(x, y, z) = xyz + x'y'z'$  and  $g_3(x, y, z) = x'y'z' + xyz + x'yz'$  are three functions with eight different output.

By using algebraic identities,

$$g_1(x, y, z) = xyz \quad (9)$$

$$g_2(x, y, z) = xyz + x'y'z' = 1 - x - y - z + xy + xz + yz \quad (10)$$

$$g_3(x, y, z) = xyz + x'y'z' + x'yz' = xyz + 1 - x - z + xz \quad (11)$$

a) From the first row we take the outputs, we have  $g_1(x, y, z) = 1, g_2(x, y, z) = 1$ , and  $g_3(x, y, z) = 1$ .

Therefore the equation becomes,

$$xyz = 1$$

$$1 - x - y - z + xy + xz + yz = 1.$$

$$xyz + 1 - x - z + xz = 1$$

$\therefore x = 1, y = 1, z = 1$  which is the corresponding input.

b) From the first row we take the outputs, we have  $g_1(x, y, z) = 0, g_2(x, y, z) = 1,$  and  $g_3(x, y, z) = 1.$

Therefore the equation becomes,

$$xyz = 0$$

$$1 - x - y - z + xy + xz + yz = 1$$

$$xyz + 1 - x - z + xz = 1$$

$$\Rightarrow x + z = xz$$

$$(i.e)x = 0, z = 0$$

$$\therefore y = 0$$

$\therefore x = 0, y = 0$  and  $z = 0$  which is the corresponding input.

c) From the first row we take the outputs, we have  $g_1(x, y, z) = 0, g_2(x, y, z) = 0,$  and  $g_3(x, y, z) = 1.$

Therefore the equation becomes,

$$xyz = 0$$

$$1 - x - y - z + xy + xz + yz = 0$$

$$xyz + 1 - x - z + xz = 1$$

$$\Rightarrow x + z = xz$$

$$(i.e)x = 0, z = 0$$

$$\therefore y = 1$$

$\therefore x = 0, y = 1$  and  $z = 0$  which is the corresponding input.

d) From the first row we take the outputs, we have  $g_1(x, y, z) = 0, g_2(x, y, z) = 0,$  and  $g_3(x, y, z) = 0.$

Therefore the equation becomes,

$$xyz = 0$$

$$1 - x - y - z + xy + xz + yz = 0$$

$$xyz + 1 - x - z + xz = 0$$

$$\Rightarrow 1 - x - z + xz = 0$$

$$\Rightarrow xy - y + yz = 0$$

$$(i.e)y(x - 1 + z) = 0$$

$$\therefore 3 \text{ cases}$$

<i>case1:</i> $y(x - 1 + z) = 0$ $y = 0, x - 1 + z = 0$ <i>(i.e)</i> $y = 0, x = 1, z = 0$ $y = 0, x = 0, z = 1$	<i>case2:</i> $y(x - 1 + z) = 0$ $y = 0, x - 1 + z = 1$ <i>(i.e)</i> $y = 0, x = 1, z = 1$	<i>case3:</i> $y(x - 1 + z) = 0$ $y = 1, x - 1 + z = 0$ <i>(i.e)</i> $y = 1, x = 1, z = 0$ $y = 1, x = 0, z = 1$
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x	Y	z	values	$g_1(x, y, z)$	$g_2(x, y, z)$	$g_3(x, y, z)$
1	1	1	$\varphi(1)$	1	1	1
0	0	0	$\varphi(2)$	0	1	1
0	1	0	$\varphi(3)$	0	0	1
1	0	0	$\varphi(4)$	0	0	0
0	0	1	$\varphi(5)$	0	0	0
1	0	1	$\varphi(6)$	0	0	0
1	1	0	$\varphi(7)$	0	0	0
0	1	1	$\varphi(8)$	0	0	0

**Possible Redefined Equations:** There are  $2^{2^n} - 2$  terms. Here  $n=3$ . (i.e) 254 functions. Therefore, we can choose any three functions to solve.

**Verification:**

x	y	z	$f_1(x,y,z)$	$f_2(x,y,z)$	$f_3(x,y,z)$
1	1	1	1	1	1
0	0	0	0	1	1
0	1	0	0	0	1
1	0	0	0	0	0
0	0	1	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
0	1	1	0	0	0

$$f_1(x, y, z) = xyz \quad (12)$$

$$f_2(x, y, z) = xyz + x'y'z' \quad (13)$$

$$f_3(x, y, z) = xyz + x'y'z' + x'yz' \quad (14)$$

The above equation is same as (9), (10) and (11).

**Solution of System of Boolean Equation to Give the Input of the Circuit (more variables):** The above method is applicable for 'n' number of variable also. But the complication arises when we solve for system of n equations. In order to avoid this, as we defined in 2.4 and 2.6 any complicated circuit can be reduced, so that we get the equivalent circuit (i.e. the variables can be reduced) and above concept is implemented on it.

## 2. CONCLUSION

In hospital and in star hotels, there is connection between the reception telephones to the room telephone. Sometimes the call received in the reception is not properly sent to the respective rooms and this problem is rectified by the above method. This paper concludes that redefined equations are same as the system of Boolean equation. With the help of that redefined equations, by solving we get the input for the given network. The thinking behind the two variables can be applied to three or more variables, so that we can solve the multi complex problem.

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