Analysis and experimental validation of asymmetrically loaded clamped circular plate

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ABSTRACT

Exact analytical expressions are available in simple case of centrally loaded clamped circular plates. As soon as the loading considered is non-central, the available expression to determine deflection, slope and curvature are no longer simple and are based on series expansion for deflections which have to be differentiated to obtain slope and curvature. This is a tedious process.

The Finite Element Method (FEM) is a numerical technique for finding approximate solution for complex loading conditions and the clamped circular plate is modelled using ANSYS. It is loaded on the diameter of the plate other than its centre as a first case. The slope and curvature contours are plotted and to validate the numerical analysis, convergence test is conducted by discretising plate into more number of elements.

In order to validate the results, an experiment is conducted on a reflective clamped circular plate for the same loading condition. The light source from He-Ne laser is passed through a transparent (bi-directional) knitted grid pattern before it falls on the reflective clamped circular plate. The reflected light that falls on the screen before and after loading the plate is captured using a CCD camera. The change is position of various intersecting points the grid enables determining slope and curvature contours for the entire plate.

KEY WORDS: asymmetrical loading, clamped circular plate, FEM, optical experiment.

1. INTRODUCTION

Plates are extensively used in many applications like roof and floor of buildings, turbine disks, water tanks, etc. Plates used in such applications are normally subjected to lateral loads causing the bending of plates. Plate bending or bending of plates refer to the deflection of plate perpendicular to plane of plate under the action of external force. The analysis of the deformation field under various loading plays the vital role in determining their failures. The exact solution is available for certain known loading condition like central loading for clamped circular plate whereas getting the exact solution for asymmetrical loading condition is hard. The objective of this work is to determine the deformation field for loaded clamped circular plate other than the centre numerically and validate it experimentally.

2. EXPERIMENTAL

Governing equation for circular plates: Circular plate governing equations can be obtained in two ways. One by using transformation relations between polar coordinates (r, θ) and rectangular coordinates (x, y), through which the expression can be expressed in polar coordinates from Cartesian coordinates. Second being direct derivation of the equation in polar coordinates.

The equation of equilibrium governing the linear bending of an isotropic circular plate:

In rectangular coordinates

$$D\left(\frac{\partial^4 w_\circ}{\partial x^4}+2\frac{\partial^4 w_\circ}{\partial x^2\partial y^2}+\frac{\partial^4 w_\circ}{\partial y^4}\right)+kw_\circ=q(x,y) \qquad (1)$$

In polar coordinates

$$D\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_\circ}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w_\circ}{\partial\theta^2}\right)+kw_\circ=q(r,\theta) \qquad (2)$$

Asymmetric bending for clamped circular plate – general equation: Non-axisymmetric loading condition is applied on circular plate, which means that the geometry and the boundary conditions are axisymmetric, only thing that changes is the loading which is not axisymmetric. Eq.1 and eq.2 are used in deriving the governing equation for asymmetrically loaded clamped circular plate.

The homogeneous solution may be expressed in the following form

$$w_h(r,\theta)=\sum_{n=0}^{\infty}a_n(r)\cos n\theta+\sum_{n=1}^{\infty}b_n(r)\sin n\theta \qquad (3)$$

Where a_n and b_n are the functions of radius (r) only. The set of equations which are functions of r have been derived individually.

$$\left.\begin{aligned}
a_0 &= A_0+B_0\,r^2+C_0\log r+D_0\,r^2\log r\\
a_1 &= A_1+B_1\,r^3+C_1\,r^{-1}+D_1\,r\log r\\
b_1 &= E_1\,r+F_1 r^3+G_1\,r^{-1}+H_1\,r\log r\\
a_n &= A_n\,r^n+B_n r^{-n}+C_n\,r^{n+2}+D_n\,r^{-n+2}\\
b_n &= E_n r^n+F_n r^{-n}+G_n\,r^{n+2}+H_n\,r^{-n+2}
\end{aligned}\right\} \qquad (4)$$

For n=2, 3… where A_n,…….,H_n are constants that are determined using boundary conditions.

The generalised form as in eq.3, being a series expression, it is difficult to apply differentiation to a series expansion in order to obtain the slope and curvature values. The process being a tedious one, Finite Element Analysis is used as an alternative.

Finite element analysis for asymmetrically loaded clamped circular plate: Finite Element Analysis gives the approximate solution for problems that arise in different domains. As there is no exact solution to find the slope and curvature for an asymmetrically loaded clamped circular plate, numerical approach using finite element method is used. In order to validate the numerical approach, method of convergence is used. A clamped circular plate is modelled in Ansys as per table 1; the required loading condition is applied and solved. The deformation values at the nodes for entire plate were extracted. The contour plot of partial slope along x and y direction for the entire plate is shown in fig.1.

Table 1: Input for modelling in Ansys

Element type	**4node SHELL181**
Radius	20mm
Young's modulus	2500 N/mm²
Poisson's ratio	0.3
UX,UY,UZ(all exterior nodes)	0
RX,RY,RZ (all exterior nodes)	0
Known displacement	20μm

Method of convergence: Convergence test is a method to verify the numerical solution for its validity. It means further refinement of mesh doesn't alter the results. The numbers of element divisions are increased in order to check whether the nodal values are converged and it is visibly seen as smooth contour curves in fig 1(b) as compared to fig. 1(a).

It is seen from fig.1 that the plate is displaced at the point of loading so that the values of slope are relatively higher than the boundaries. At the boundaries of the plate the values of slope obtained are zero indicating the clamped circular plate condition.

Experimental validation for asymmetrically loaded clamped circular plate: Even though method of convergence is used to verify numerical solution, experimental validation is necessary and thus an optical experiment is conducted on a reflective asymmetrically loaded clamped circular plate as shown in fig.2.

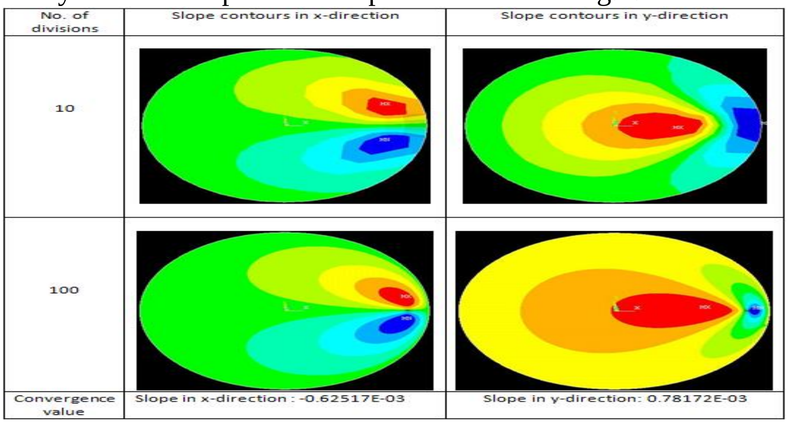

Figure.1. Convergence of slope in x and y directions

A laser light is allowed to pass through collimating lens to and fall on to a specularly reflecting clamped circular plate of 40mm diameter and 2mm thick. Before reaching the plate surface the light is allowed to pass through a set of horizontal and vertical grid lines. The reflected light is allowed to fall on to a white screen and a CCD camera is used to capture the image from the screen. Images are captured for both loaded and unloaded conditions as shown in fig.3. The shift in coordinate location at the point of intersection of grid lines is used as a factor in determining the slope value.

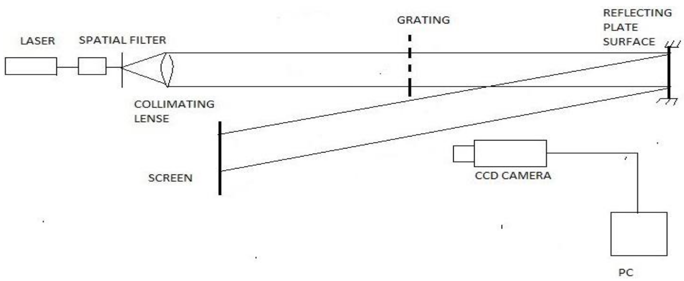

Figure.2. Experimental setup

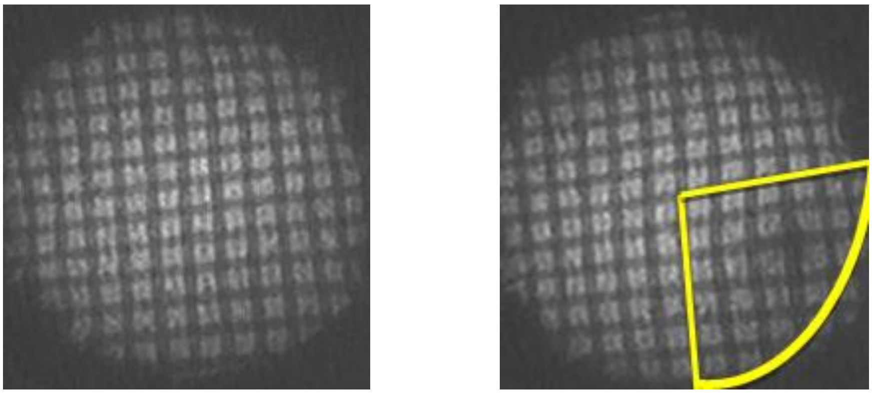

Figure 3: unloaded and asymmetrically loaded specimens

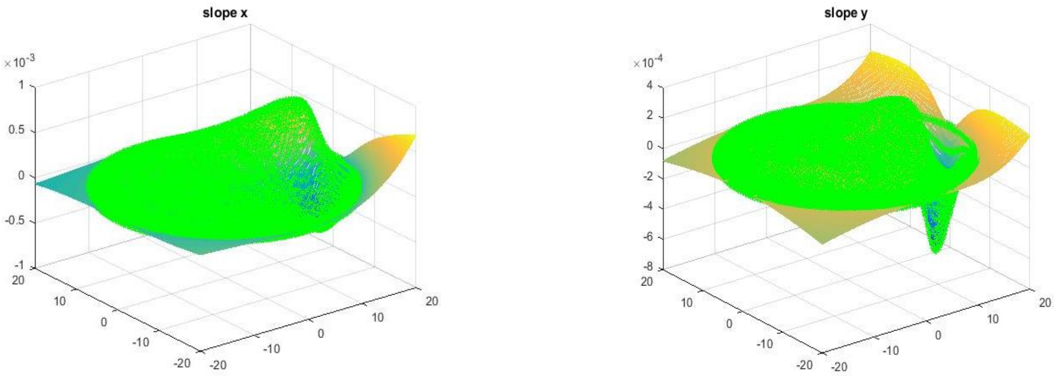

Figure 4: surfaces developed using slope values obtained from numerical analysis

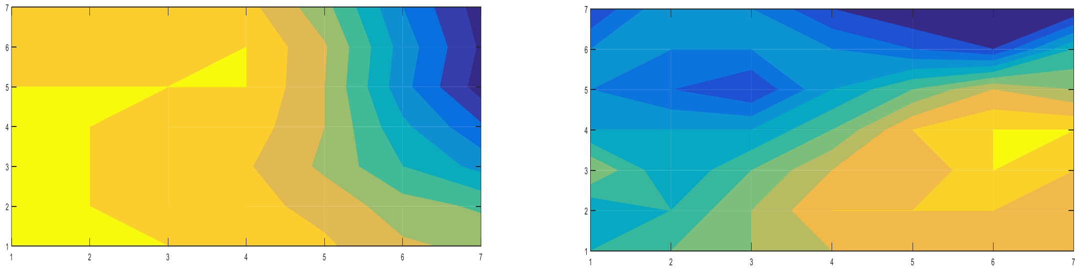

Figure 5: slope contours developed for selected portion of the image in x and y direction respectively

3. RESULTS AND DISCUSSIONS

Slope values along with node locations are extracted as an output from numerical analysis. These values are imported into Matlab.

The images obtained from experiment are related to the actual specimen so that the coordinate location of the image matches with those of the specimen through a scalar magnification factor. Since, the slope values determined from Ansys are directly related to the specimen. There exists a one to one mapping between the specimen, image and Ansys values

Interpolation of gridded data: In Matlab the slope values and node location are imported and a surface is developed using the scattered data available from ansys as shown in fig.4. The specimen spatial coordinates are formed in to a gridded pattern at regular intervals and the values of slope are interpolated.

The experimental data is computed by considering the shift in pixel location at the intersection points of gridded pattern and slope values are computed. Also slope contour are developed using Matlab.

The slope contours developed with difference in location are represented in fig.5. These contours are similar to that developed using Ansys as shown in fig.2, at the region of interest

4. CONCLUSIONS

For asymmetrically loaded clamped circular numerical analysis is the only alternative available. Method of convergence and experimental validation were used to validate the numerical analysis an error of 17.8% was obtained when compared. By considering fine grid, more number of intersection points can be used for the study there by reducing the percentage of error.

Future scope: Asymmetrical load can be applied at random locations and values at desired locations can be extracted.

Here, concept of reflection has been employed and experiment has been conducted. With slight modification in experimental setup the experiment can be conducted using concept of transmission and the results can be compared.

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