

Tuning of IMC based fractional order PID controller for level control in spherical tank

Kirthini Godweena A and K Sundaravadivu

Dept. of Electronics & Instrumentation Engineering, St. Joseph's College of Engineering, Chennai-600119.

*Corresponding author: E.Mail: kirthini.888@gmail.com

ABSTRACT

This paper proposes a tuning method for fractional controllers based on Internal Model Control for a fractional controller designed for a first order Spherical tank. Modelling of the spherical tank has been done by obtaining the transfer functions at four different operating points. With these, the controllers are designed, simulated and the real time implementation is carried out by applying the controllers to the hardware setup. IAE values are calculated and tabulated in comparison to the performance rendered by integer order PID controllers. Here the Spherical tank is chosen for the analysis as it is one of the best and basic examples to study control and its effects.

Keywords: IMC, FOPID, Spherical tank, $PI^{\lambda}D^{\mu}$, IAE

1. INTRODUCTION

Fractional calculus is the combination of integration and differentiation to non-integer orders. It has come down from ordinary calculus by extending ordinary differential equations (ODE) to fractional-order differential equations (FODE). Therefore, a fractional-order proportional – integral - derivative (FOPID) controller is a variation of a standard (integer) PID controller. The output of an FOPID is a linear combination of the input, a fractional derivative of the input and a fractional integral of the input. Usually the PID controller is the most famous in the industry with its due simplicity, performance robustness and availability of many effective simple tuning methods based on minimal plant knowledge. Unlike integer order PID controllers, the fractional derivatives provide an excellent device for the description of inherent and memory properties of various processes and materials. This is the main advantage of fractional derivatives in contrast with classic integer-order models, in which such effects are eliminated. The advantages of fractional derivatives become vivid in modeling mechanical and electrical properties of real materials, as well as in the description of material properties of rocks, and in many other fields. Fractional integrals and derivatives are applied in control of dynamical systems theoretically, when a fractional differential equation describes the controlled system or/and the controller. The mathematical modeling and simulation of systems and processes, based on their properties in terms of fractional derivatives, naturally leads to fractional order differential equations and gives rise, the necessity to solve such equations.

In this paper, a class of fractional order controller (PID^{μ}) is designed and tuned based on Internal Model control (IMC) for a first order Spherical tank. This has been done in comparison to integer order controllers based on IMC rule.

2. METHODOLOGY

The Internal Model control philosophy (IMC) is based on the Internal model principle, which states that control can be achieved only if the control system envelops, either implicitly or explicitly, some representation of the process to be controlled. If the control method is developed based on an exact modeling of the process, then perfect control of the process is theoretically possible. The IMC philosophy is applied here to generate settings for conventional PID controllers as shown in Figure 1.

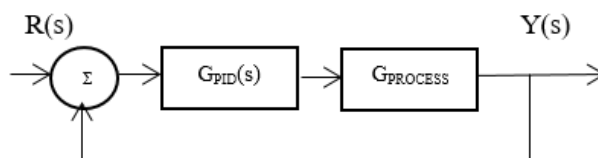


Figure.1. IMC-PID Control scheme

$$\text{Thus } G_{PID}(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s) \tilde{G}_p(s)} \quad (1)$$

$$\text{where } G_{PID} = \tilde{G}_p^{-1}(s), G_{IMC} = \frac{\tilde{G}_p^{-1}(s) G_f(s)}{1 - \tilde{G}_p^{-1}(s) G_f(s)} \quad (1a)$$

[G_p - process transfer function, G_r -filter transfer function]

Fractional calculus comprises of fractional integration and differentiation. Therefore a generalized integral and differential operator has been combined into a single fundamental operator represented by ${}_a D_t^{\lambda}$ where a and t are

the limits and λ ($\lambda \in \mathbf{R}$) is the order of the operation. For positive λ , it denotes derivative and negative λ , it does integral action as

$${}_a D_t^\lambda = \begin{cases} \frac{d^\lambda}{dt^\lambda} & \text{Real}(\lambda) > 0 \\ 1 & \text{Real}(\lambda) = 0 \\ \int_a^t (d\tau)^{-\lambda} & \text{Real}(\lambda) < 0 \end{cases} \quad (2)$$

Several methods are available to define fractional-order derivatives and integrals. The definitions used in vogue for fractional derivatives are Riemann-Liouville, Grunwald-Letnikov and Caputo definitions [17]. The Caputo fractional derivative of order λ with respect to the variable t is defined as

$${}_a D_t^\lambda = \begin{cases} \frac{1}{\Gamma(n-\lambda)} \int_0^t \frac{f^n(\tau) d\tau}{(t-\tau)^{\lambda+1-n}}, n-1 < \lambda < n \\ \frac{d^n}{dt^n} f(t), \lambda = n \end{cases} \quad (3)$$

where n is the first integer not less than λ , $\Gamma(z)$ is Euler's Gamma function which is given by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt. \quad (3a)$$

The Laplace transform of the Caputo fractional derivative;

$$L\{{}_0 D_t^\lambda f(t)\} = s^\lambda F(s) - \sum_{k=0}^{n-1} s^{\lambda-1-k} f^{(k)}(0) \quad (4)$$

where $n-1 < \lambda \leq n \in \mathbf{N}$.

The main advantage of using the Caputo definition is that, only integer order derivatives of function $f(t)$ at $t=0$ appear in the Laplace transform of the Caputo fractional derivative. For zero initial conditions in the above equation, a straightforward result is obtained as $L\{{}_0 D_t^\lambda f(t)\} = s^\lambda F(s)$ (4a)

Tuning based on IMC: Let us assume that controller in the block diagram in Fig. 1 is of the form

$$C(s) = \frac{C_c(s)}{1 - C_c(s) \tilde{G}_p(s)} \quad (5)$$

where $\tilde{G}_{PID}(s)$ is assumed as $C(s)$. It is observed that for a delayed system, $C(s)$ has a simple pole at $s=0$. Considering this, if $C(s)$ is rewritten as

$$C(s) = \frac{f(s)}{s} \quad (6)$$

For tuning a specified controller, it must possess a simple pole at origin. Here it is assumed $\tilde{G}_p(s) = G_p(s)$ (7)

Controller structures that are used for tuning are a well-known category of PID and a formulation fractional order PID controller as follows,

$$C_i(s) = k_p \left(1 + \frac{1}{k_i s} \right) \left(\frac{1 + k_d s}{1 + a k_d s} \right) \quad (8)$$

$$C_f(s) = k_p \left(1 + \frac{k_i}{s} \right) (1 + k_d s)^m$$

Spherical tank: Identification of the system refers to the determination of the system parameters like time constants, dead time & system gain of the respective models. Among various types of models, FOPDT and SOPDT models are widely used to describe the dynamics of many real time systems. In this project the spherical tank is modelled as an FOPDT system. The volume of the sphere is given by,

$$V = \frac{4}{3} \pi R^3 \quad (9)$$

The mass balance equation is,

$$F_{in} - F_{out} = \frac{dV}{dt} \quad (10)$$

$$F_{in} - F_{out} = \pi R^2 \left[1 - \frac{(R-h)^2}{R^2} \right] \frac{dh}{dt} \quad (11)$$

where h – height of liquid in tank in cm

R – radius of the sphere in cm

F_{in} – inlet flow rate in lph

F_{out} – outlet flow rate in lph

Applying the steady state values and solving equations (8) to (10), the following transfer function model of the spherical tank is obtained

$$G_p(s) = \frac{K_p e^{-\theta_d s}}{\tau s + 1} \quad (12)$$

where K_p – system gain = $\frac{2h_s}{F_{out,s}}$ (12a)

h_s – height of liquid level in the tank at steady state

$F_{out,s}$ – outlet flow rate at steady state

θ_d – dead time

τ – time constant = $4\pi K_p h_s$

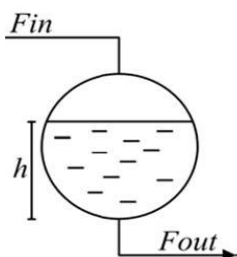


Figure.1. Modelling of Spherical tank

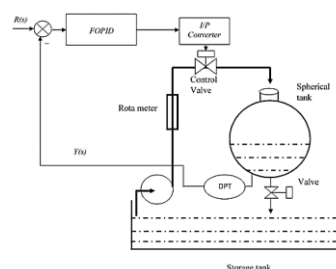


Figure.2. Experimental setup of Spherical tank

Table.1. Specifications of the experimental setup of spherical tank

| Sections | Specifications |
|--------------------|--|
| Spherical Tank | Diameter and Height: 50cm made up of stainless steel |
| Level Transmitter | Differential Pressure Transmitter |
| Pump | Centrifugal Pump |
| Control Valves | Pneumatic Valves |
| Rotameter | (0-1000) lph |
| I/V, V/I Converter | (4-20) mA to (0-5) V and vice versa |
| I/P Converter | (4-20) mA to (3-15) psi |
| Interface | Interfacing module (MATLAB compatible) connecting system to PC |

Implementation:

Integer tuning rule: As it is seen from the equation (8), $C_i(s)$ can be written in the form of

$$C_i(s) = \frac{g(s)}{s(1 + \beta s)} \quad (13)$$

Now it is obvious that $f(s)$ is only a function of system parameters. To determine a value for β from, one can guess that $k_d = 0.25 \max(1, t_p)$, where l is time delay and t_p is slow pole of system and $a=0.1$ is chosen. Thus in (8), $(1 + \beta s)f(s)$ is known. To find out the values of three unknown parameters k_p , k_i , k_d , use of Taylor series gives the values of unknowns. Results have been consolidated as follows.

$$\begin{aligned} \frac{k_p}{k_i} &= g(0) = f(0) \\ k_c &= g'(0) = f'(0) + \beta f(0) \\ k_c k_d &= g''(0) = \frac{f''(0) + 2\beta f'(0)}{2} \end{aligned} \quad (14)$$

The above PID tuning formulation has some demerits that are as follows,

- It is required to assume the value of β to assist the value of k_d and again in the tuning rules the value of k_d is found.
- It is needed to define the low pass filter $F(s)$ in a complex form, which makes it a tedious job to find the design of the filter.
- There is only one set of solutions for the above equations, which makes achieving robust stability difficult. With fractional rule definition, as given below these disadvantages can be eliminated.

Fractional order tuning rule: Considering,

$$C_f(s) = k_p \left(1 + \frac{k_i}{s}\right) (1 + k_d s)^m \quad (8)$$

there are four unknowns to be determined. To find these parameters, $C_F(s)$ should be rewritten as $C_F(s) = \frac{g(s)}{s}$ (15)

$$\text{where } g(s) = k_p (s + k_i)(1 + k_d s)^m. \quad (16)$$

On the other hand, $C_F(s)$ can also be defined as function of system parameters. From relation $f(s)=g(s)$, where $f(s)$ and $g(s)$ are defined previously. It is possible to figure the unknowns k_p , k_i , k_d , m . this tuning can be done by Taylor's series as done below,

$$g(s) = g(0) + g'(0) + \frac{g''(0)}{2!} s^2 + \frac{g'''(0)}{3!} s^3 + \dots$$

$$f(s) = f(0) + f'(0) + \frac{f''(0)}{2!} s^2 + \frac{f'''(0)}{3!} s^3 + \dots \quad (17)$$

$$g(s) = k_p (s + k_i)(1 + k_d s)^m$$

Because of four unknowns, one way to tune is to use the first four Taylor series terms of $g(s)$ and $f(s)$. This results in the following equations,

$$g(0) = k_p k_i$$

$$g'(0) = k_p + (k_p k_i)(m k_d) \quad (18)$$

$$g''(0) = 2(k_p (m k_d)) + (m^2 k_d^2 - m k_d^2)$$

$$g'''(0) = 3(k_p (m^2 k_d^2 - m k_d^2)) + k_p k_i (m^3 k_d^3 - 3m^2 k_d^3 + 2m k_d^3)$$

To compute solutions for this set of nonlinear equations and four unknowns, the following steps are carried out,

$$C_1 = (k_p k_i) = f(0)$$

$$C_a = (m k_d) = \frac{f'(0) - k_p}{f(0)} \quad (19)$$

By replacing k_p , k_i , k_d , m with their above equivalent terms in (14), the following reduces equations are obtained,

$$C_a (2k_p + C_1(-k_d + C_a)) = f''(0) \quad (20)$$

$$3k_p C_a (-k_d + C_a) + C_1 (C_a^3 - 3k_d C_a^2 + 2k_d^2 C_a) = f'''(0)$$

From the set of 2 equations-2 unknowns, k_d , k_p can be obtained. From the first equation of (20), k_d is defined, then substituting in the second relation, it gives rise to a fourth order equation of k_p . Thus there will be four choices to define k_p . These choices make up for a more powerful robust stability design.

Defining low pass filter: The rules for defining the filter are the same as that specified in references [8,13].

NINTEGER toolbox: Ninteger is a toolbox used in MatLab that is used to develop fractional order controllers for single-input, single-output plants to observe and assess their performance. It was developed because formulating controllers based on fractional calculus requires running several algorithms which are not found in toolboxes distributed with Matlab or in any easily available toolbox. The code can be freely used and altered as per the requirements. Two functions namely nid and nipid are used in vogue. This paper uses nipid function in the model. Many of these algorithms concern building integer order plants that are approximately identical to the behavioural characteristics of fractional order systems.

Identification of FOPDT models: The drain valve of the spherical tank is kept constant at a particular position and calibrated from (0-1000) lph. This is not changed until the end of the experiment. Now the flow is adjusted by viewing it in the rotameter. The setpoint is set to be at 18cm. The process curve is noted at four different flow speeds.

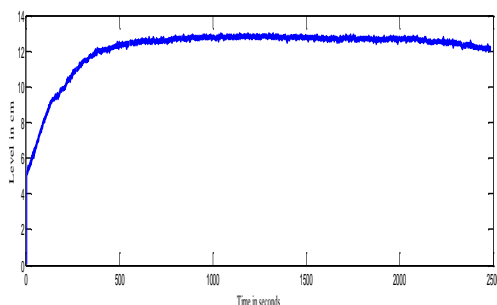


Figure.3. Open loop response at 300lph

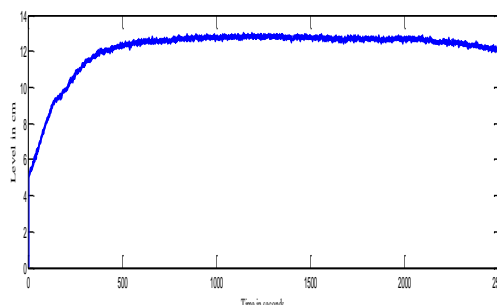


Figure.4. Open loop response at 510 lph

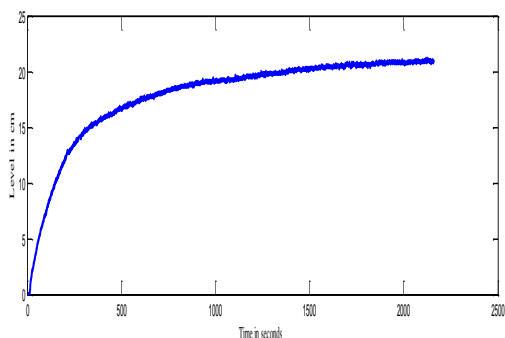


Figure.5.Open loop response at 770 lph

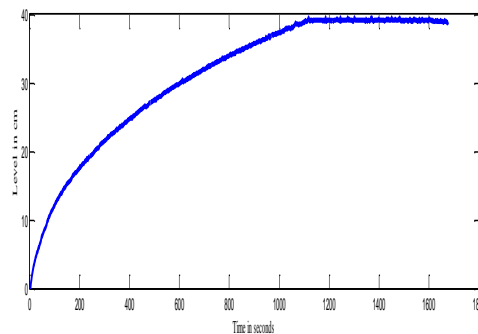


Figure.6.Open loop response at 890 lph

Table.2.Transfer functions

| Operating speed | Transfer function |
|-----------------|-------------------------------------|
| 300 lph | $\frac{0.7e^{0.5s}}{21s + 1}$ |
| 510 lph | $\frac{0.671e^{0.5s}}{20.302s + 1}$ |
| 770 lph | $\frac{4.882e^{0.5s}}{0.29s + 1}$ |
| 890 lph | $\frac{1.166e^{0.5s}}{10.934s + 1}$ |

3. RESULTS AND DISCUSSION

Simulation Results:

Operating point 1: For the transfer function 1 the values for integer order and fractional order controllers are tabulated as follows. The steps to get the control parameters are as follows; After applying the IMC rule the transfer function becomes,

$$C(s) = \frac{21s + 1}{2.94}$$

$$\frac{k_p}{k_i} = 0.3401$$

(21)

$$k_p + 0.3401k_d = 7.1429$$

$$k_p k_d = 0$$

$$C_1 = k_p k_i = 0.3401$$

$$C_a = m k_d = \frac{7.1429 - k_p}{0.3401}$$

Table.3.Values for IOPID and FOPID controllers

| IOPID | FOPID |
|-----------------------------|---|
| $k_p = 7.1429, k_i = 2.943$ | $k_p = 5.8476, k_i = 0.0581, k_d = 45.982, m = 0.082$ |
| IAE = 5.252 | IAE = 2.021 |

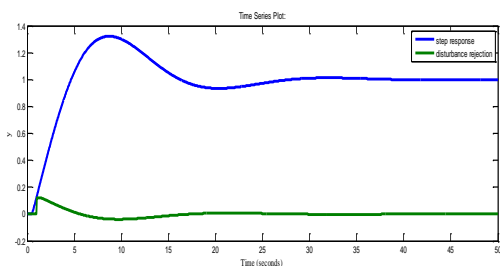


Figure.7.Step response and load disturbance response of integer order controller

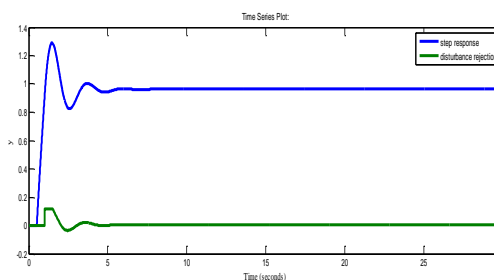


Figure.8.Step response and load disturbance response of fractional order controller

Real Time Implementation Results:

Operating point 1:

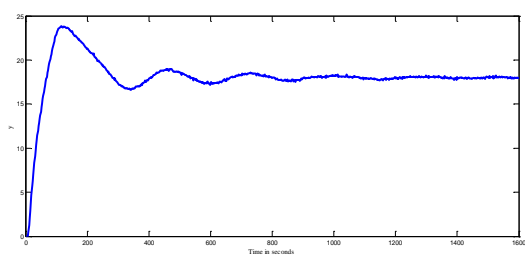


Figure.9.Closed loop response with IOPID controller

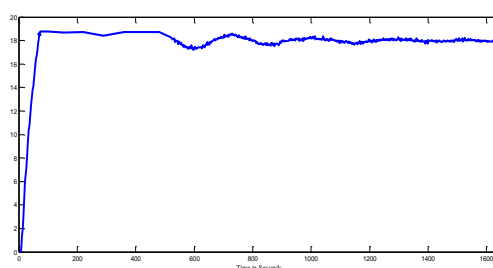


Figure.10.Closed loop response with FOPID controller

Operating point 2:

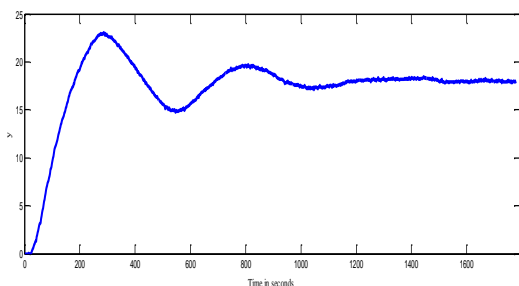


Figure.11.Closed loop response with IOPID controller

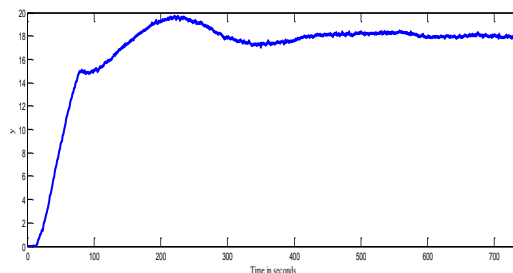


Figure.12.Closed loop response with FOPID controller

Operating point 3:

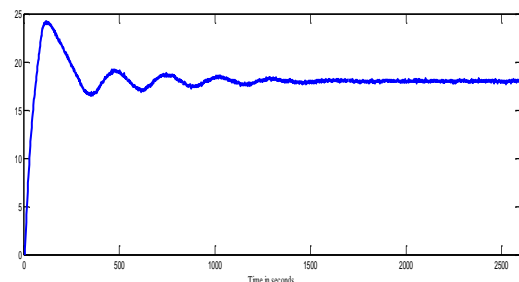


Figure.13.Closed loop response with IOPID controller

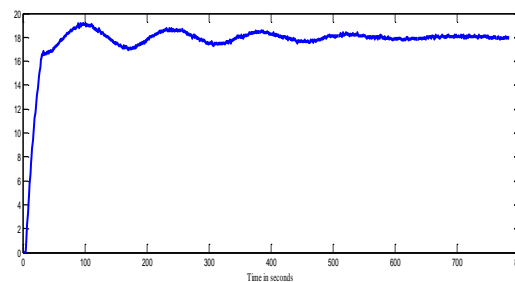


Figure.14.Closed loop response with FOPID controller

Operating point 4:

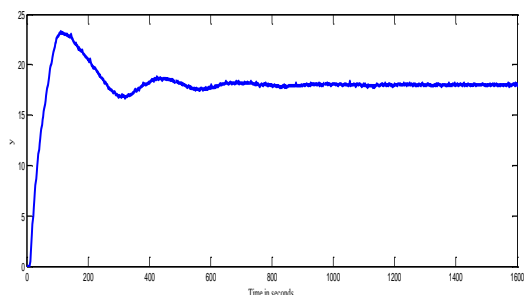


Figure.15.Closed loop response with IOPID controller

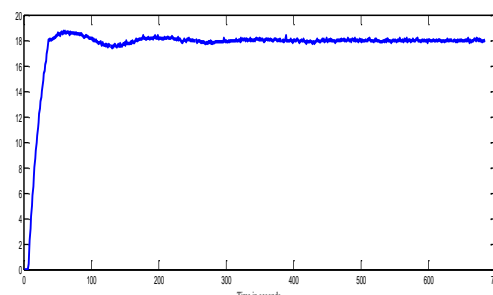


Figure.16.Closed loop response with FOPID controller

The following table is a careful consolidation of all the controller values and shows how the IAE values have been minimized with the use of fractional order PID controllers in contrast with the IOPID controllers. Their introduction advocates better performances which is clarified both by the simulation results and the real time results.

Table.4.Tabulation of controller and performance and values for IOPID

| Operating speed | Transfer Function | IOPID | | |
|-----------------|-----------------------------------|--------|--------|-------|
| | | kp | ki | IAE |
| 300 lph | $\frac{0.7e^{0.5s}}{21s+1}$ | 7.1429 | 2.943 | 5.252 |
| 510 lph | $\frac{0.671e^{0.5s}}{20.302s+1}$ | 7.462 | 2.9293 | 4.062 |
| 770 lph | $\frac{4.882e^{0.5s}}{0.29s+1}$ | 0.071 | 0.295 | 1.087 |
| 890 lph | $\frac{1.166e^{0.5s}}{10.934s+1}$ | 4.289 | 13.03 | 3.768 |

Table.5.Tabulation of controller and performance and values for FOPID

| Operating speed | Transfer Function | FOPID | | | | |
|-----------------|-----------------------------------|--------|--------|--------|-------|-------|
| | | kp | ki | kd | m | IAE |
| 300 lph | $\frac{0.7e^{0.5s}}{21s+1}$ | 5.8476 | 0.0581 | 45.982 | 0.082 | 2.021 |
| 510 lph | $\frac{0.671e^{0.5s}}{20.302s+1}$ | 6.8476 | 0.0381 | 32.982 | 0.076 | 1.341 |
| 770 lph | $\frac{4.882e^{0.5s}}{0.29s+1}$ | 0.4292 | 0.4292 | 0.7085 | 0.162 | 0.955 |
| 890 lph | $\frac{1.166e^{0.5s}}{10.934s+1}$ | 7.3223 | 0.0535 | 0.0187 | 0.999 | 2.592 |

4. CONCLUSION

The tuning rules for the fractional order controllers are formulated depending on Internal Model control and fractional calculus. The effectiveness of the procedure is observed through the various step and load disturbance responses, robust analysis for the transfer functions obtained at different operating speeds.

From the obtained results, one can conclude that the proposed controller tuning procedure can be implemented effectively without the burdening process of approximating the time delay of the model. The work can be extended for other tank models, thus elaborating on its performance

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