Non spit two out degree equitable domination number in graphs

K.Mathevan Pillai[1], M.S.Mahesh[1], G.Selvam[2]
[1]Department of mathamatics, Francis Xaviour Engineering College, Tirunelveli
[2]Department of mathematics, VMKV Engineering college, Salem
***Corresponding author: E-Mail: mathevanntc@yahoo.com**

ABSTRACT

Let G=(V,E) be a undirected graph. A dominating D of G is said to two out degree equitable dominating set if for any two vertices u,vϵ D such that $|od_D(u) - od_D(v)| \leq 2$ where $od_D(u) = |\ N(v) \cap V - D|$. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is donated by $\gamma_{2oe}(G)$. In this paper non split two out degree equitable domination in graph is introduced and studied.

AMS Classification: 05C9

KEY WORDS: two- out degree, equitable, non-split, dominating set.

1. INTRODUCTION

The graphs considered her are nontrivial, connected, simple finite and undirected. For a graph G=(V,E), V denoted it vertex set and E its edge set. The number of vertices and edges are denoted by p and q respectively. The open neighborhood of v is denoted by N(u) and defined as N(u)= {v/uvϵ E}. The out degree of v with repsect to D is denoted by $od_D(u) = |\ N(v) \cap V - D|$

The concept of domination was first studied by Ore (Kulli and Jankiram, 2000) and Berge (1962). A set D $\subseteq V$ is said to be a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The cardinality of a minimum dominating set D is called the domination number of f and is denotedby $\gamma(G)$. Ali Sahal and V.Mathad (Sahal, 2013) introduce the concept of two out degree equitable domination in graphs. A dominating D of G is said to two out degree equitable dominating set if for any two vertices u,vϵ D such that $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is donated by $\gamma_{2oe}(G)$. M.S.Mahesh and P.Namasivayam (Mahesh, 2014) introduced the concept of connected two out degree equitable domination in graphs. A two out degree equitable dominating set is called connected two out degree equitable dominating set if the induced sub graph <D> is connected. The minimum cardinality of connected two out degree equitable dominating set is called connected two out degree equitable domination number and it is donated by $\gamma_{c2oe}(G)$.V.Kulli and Jankiram (2000) introduced the concept of non-spit domination in graphs. A dominating set of V(G) is a non-spit dominating set if the induced sub graph <V-D> is connected. The minimum cardinality of non-spit two out degree equitable dominating set is called non split two out degree equitable domination number and denoted by γ_{ns}(G).

The purpose of the paper to introduce the concept of non-spit two out degree equitable domination in graphs.

Non-Split two out degree equitable domination in graphs:

Definition: A two out degree equitable dominating set D of a graph is non-split two out degree equitable dominating set if the induced sub graph<V-D> is connected. The minimum cardinality of non-split two out degree equitable dominating set is called non-split two out degree equitable domination number and it is denoted by γ_{ns2oe}(G).

Example:

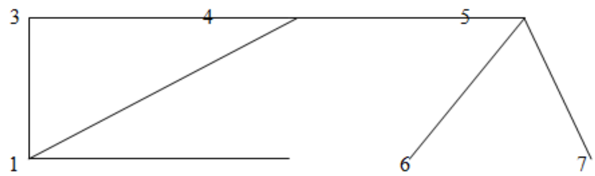

Figure.1.

Let us consider a set D = {1, 3, 6, 7} and V-D= {2, 4, 5}

$od_D(1) = |N(1) \cap \{2,4,5\}| = 1$

$od_D(3) = |N(3) \cap \{2,4,5\}| = 2$

$od_D(6) = |N(6) \cap \{2,4,5\}| = 1$

$od_D(7) = |N(7) \cap \{2,4,5\}| = 1$

Then Clearly for any u,vϵ {1,3,67} such that $|od_D(u) - od_D(v)| \leq 2$.

D= {1, 3, 6, 7} is two out degree equitable dominating set

Clearly <2,4,5> is connected

D={1,3,6,7} is minimum non-split two out degree equitable dominating set

$\gamma_{ns2oe}(G)$=4.

Observation: For any graph G with p vertices, 2$\leq$ $\gamma_{ns2oe}(G) \leq p - 2$

Non-Split two out degree equitable domination number for different graphs:

Theorem: For any complete graph: $\gamma_{ns2oe}(k_p) = 2$

Proof:

Let V=$\{u_1, u_2 - - - - - u_p\}$ be the vertices set of k_p

Let D= $\{u_1, u_2\}$ be a dominating set of G and $V - D$=$\{u_3, u_4 - - - - - u_p\}$

Now, $u_1 \epsilon D$ then $od_D(u_1) = |N(u_i) \cap V - D|$

= $|\{u_2, u_3, u_4 - - - - - u_p\} \cap \{u_3, u_4 - - - - - u_p\}|$

= $|\{u_3, u_4 - - - - - u_p\}|$= $p - 2$

Similarly $od_D(u_2) = p - 2$

Then $|od_D(u_1) - od_D(u_2)| \leq 2$. For any $u_i, u_j \epsilon\ D$

So D is two out degree equitable dominating set

The induced sub graph$< V - D >$ is connected

$\gamma_{ns2oe}(k_p) \leq 2$and 2$\leq \gamma_{ns2oe}(k_p)$

Then $\gamma_{ns2oe}(k_p) = 2$

Theorem: If G is a star$k_{1,p}$, then $\gamma_{ns2oe}(k_{1,p}) = p - 2$

Proof:

Let V=$\{v, u_1, u_2 - - - - - u_p\}$ be the vertices set of $k_{1,p}$

Let D= $\{u_1, u_2 - - - - - u_{p-2}, u_{p-1}\}$ be a domination set of G and $V - D$= $\{v, u_p\}$

Now, $u_i \epsilon D$ then $od_D(u_i) = |N(u_i) \cap V - D|$

= $|\{v\} \cap \{v, u_p\}|$

= $|\{v\}|$=1

Then $|od_D(v) - od_D(u_i)| \leq 2$. For any $u_i \epsilon\ D$

So D is two out degree equitable dominating set and $< V - D = \{v, u_p\} >$ is connected

D be minimal non-split two out degree equitable dominating set

$then\ \gamma_{ns2oe}(k_{1,p}) = p - 2$

Theorem: For any complete bipartite graph$k_{s,t}$, is $\gamma_{ns2oe}(k_{s,t}) = \begin{cases} 2 & if\ |s - t| \leq 2 \\ s + t - 2 & otherwise \end{cases}$

Proof:

Let V=$\{u_1, u_2, u_3 - - - - - u_s, v_1, v_2, v_3, - - - - - v_t\}$ be the vertices set of $k_{m,n}$ and $\{u_1, u_2, u_3 - - - - - u_s\}$ and $\{v_1, v_2, v_3, - - - - - v_t\}$ be the partition of V.

Case(i) $|s - t| \leq 2$

Let D=$\{u_i, v_j\}$ be a dominating set of G and

V-D=$\{u_1, u_2, u_3 - - - u_{i-1}, u_{i+1} - - - - u_s, v_1, v_2, v_3, - - - - v_{j-1}, v_{j+1} - - - v_t\}$

Now, $u_i \epsilon D$ then $od_D(u_i) = |N(u_i) \cap V - D|$

=$|\{v_1, v_2, v_3, - - - - v_{j-1}, v_{j+1} - - - v_t\} - \{u_1, u_2, u_3 - - - u_{i-1}, u_{i+1} - - - - u_s, v_1, v_2, v_3, - - - - v_{j-1}, v_{j+1} - - - v_t\}$

= $|\{v_1, v_2, v_3, - - - - v_{j-1}, v_{j+1} - - - v_t\}|$= $t - 1$

$if\ v_j \epsilon D$then$od_D(v_j) = |N(v_j) \cap V - D|$

=$|\{u_1, u_2, u_3 - - - u_{i-1}, u_{i+1} - - - - u_s\} \cap \{u_1, u_2, u_3 - - - u_{i-1}, u_{i+1} - - - - u_s, v_1, v_2, v_3, - - - - v_{j-1}, v_{j+1} - - - v_t\}$

=$|\{u_1, u_2, u_3 - - - u_{i-1}, u_{i+1} - - - - u_s\}|$=$s - 1$

$|od_D(u_i) - od_D(v_j)| = t - 1 - s + 1 = t - s \leq 2$

Then$|od_D(u_i) - od_D(v_j)| \leq 2$. For any $u_i, v_j \epsilon\ D$

So D is two out degree equitable dominating set and the induced sub graph$< V - D >$ is connected

$\gamma_{ns2oe}(k_{s,t}) \leq 2$and 2$\leq \gamma_{ns2oe}(k_{s,t})$

$\gamma_{ns2oe}(k_{s,t}) = 2$

Case (ii) $|s - t| \geq 2$ and s,t ≥ 2

Let D=$\{u_1, u_2, u_3 - - - - - u_{s-1}, v_1, v_2, v_3, - - - - - v_{t-1}\}$ be the vertices set of $k_{s,t}$ and $V - D$=$\{u_s, u_t\}$

Clearly $< V - D >$ is non-split two out degree equitable dominating set

Then $\gamma_{ns2oe}(k_{s,t}) \leq s + t - 2$. and $s + t - 2 \leq \gamma_{ns2oe}(k_{s,t})$

Then $\gamma_{ns2oe}(k_{s,t}) \leq s + t - 2$

Theorem: For any cycle C_P, then $\gamma_{ns2oe}(C_P) = p - 2$

Proof:

Let V=$\{u_1, u_2 - - - - - u_p\}$ be the vertices set of C_P
Let D=$\{u_1, u_2 - - - - - u_{i-1}, u_{i+2}, - - - - u_p\}$ dominating set of $C_P$and $V - D = \{u_i, u_{i+1}\}$
Now $od_D(u_j) = 0$, j=1,2-----------$i-2$, $i+3$,--------p
$od_D(u_i) = 1$ and $od_D(u_{i+1}) = 1$
Then $|od_D(u_i) - od_D(u_j)| \leq 2$
Then D is two out degree equitable dominating set
So $\gamma_{2oe}(C_P) = p - 2$, and $< V - D >$ is connected
Then D is minimum non split two out degree equitable dominating set
$\gamma_{ns2oe}(C_P) = p - 2$
Theorem: For the Path P_p, $\gamma_{ns2oe}(P_p)$ =$p-2$. p≥ 2
Proof:
Since the degree of any vertex in P_p is 2 except the initial and terminal vertices.
Let D=$\{v_1, v_2, v_3 - - - - - v_{p-2}\}$ and $V - D = \{v_{P-1}, v_P\}$
Clearly D is two out degree equitable dominating set
Then $< V - D >$ is connected
So D is non-split two out degree equitable dominating set
Then $\gamma_{ns2oe}(P_p)$ =$p-2$
Theorem: For the WheelW_p, : $\gamma_{ns2oe}(W_p) = \begin{cases} 2 & if\ p = 4,5 \\ p-4 & if\ p \geq 7 \end{cases}$
Proof:
Let W_p be a when with $p-1$ vertices on the cycle and a single vertex at the center. Let V (W_p) = {v,v_1,v_2,v_3----------v_{p-1}}, where u is the center and v_i (1≤ $i \leq p-1$) is on the cycle. Clearly deg (v_i) = 3 for all 1≤ $i \leq p-1$ and deg (u) =$p-1$.
Clearly p≥ 4. We have the following cases
Case 1. p=4 and 5
If p=4 then W_4 forms a complete graph then by theorem 3.1 $\gamma_{ns2oe}(W_4) = 2$
If p=5. Let us take D= {u,v_i} and $V - D = \{v_1, v_2, - - - - v_{i-1}, v_{i+1} - - - - v_{p-1}\}$ since u is adjacent with v_i for all i 1≤ $i \leq 4$, $V - D \subset$ N (u) so N(u)∩ $V - D \subset V - D$
$od_D(u)$= $|N$(u)∩ $V - D| = |\ V - D|$=3
Now for v_i ,Since deg (v_i) =3 and v_i is adjacent to u∈D then N (v_i) = {u,v_j,v_k}
and $N(v_i)$∩ $V - D$= {v_j,v_k}
$od_D(v_i)$= $|N$(u)∩ $V - D| = 2$
$|od_D(u) - od_D(v_i)| = 1 \leq 2$and clearly < $V - D$>is connected
Hence $\gamma_{ns2oe}(W_p)$ ≤2 and 2≤ $\gamma_{ns2oe}(G)$
Hence $\gamma_{ns2oe}(W_p)$ =2
Case 2.p≥6
In this case deg (u) =p, while deg (v_i) =3 for all i, 1≤ $i \leq 5$,
Let us take D={ u,v_1,v_2,v_3----------v_{m-4},} be a dominating set and $V - D = \{v_{p-3}, v_{p-2,} v_{p-1,} v_p\}$ since u is adjacent with v_i for all i, $V - D \subset$ N(u)
so$N$(u)∩ $V - D \subset V - D$
$od_D(u)$= $|N$(u)∩ $V - D| = 4$
Now for v_i and v_j
If v_i and v_j is adjacent N(v_i)= {u,v_j,v_k} and N(u)∩ $V - D$= {v_k}
$od_D(v_i)$= $|N$(u)∩ $V - D| = 1$
If v_i and v_j are not adjacent but v_i and v_j are adjacent with u so N(u)∩ V-D contains two elements so $od_D(v_i)$= $|N$(u)∩ $V - D| = 2$
So for any elements u,v∈ D
$|od_D(u) - od_D(v)| \leq 2$and clearly < $V - D$>= is connected
So D is non split two out degree equitable dominating set
Hence $\gamma_{ns2oe}(W_p)$=$p-4$
Theorem: For the double star$S_{r,t}$, : $\gamma_{ns2oe}(S_{r,t})$= $r + t$
Proof:
Let$\{u, u_1, u_2, u_3 - - - u_r, v, v_1, v_2 v_3 - - - - - v_t\}$ are the vertices of $S_{r,t}$ and all u_i is adjacent to u and v_i is adjacent to v.

Here $\{u_1, u_2, u_3 - - - - u_r, v_1, v_2 v_3 - - - - - v_t\}$ be the isolated vertices and D={u,v} is support vertices and is connected

So let us take D=$\{u_1, u_2, u_3 - - - - u_r, v_1, v_2 v_3 - - - - - v_t\}$ and $V - D$ ={u,v}

Since every vertices has one neighborhood so clearly D is two out degree equitable dominating set

Clearly $< V - D >$ is connected

$\gamma_{ns2oe}(S_{r,t}) = r + t$

Theorem: For any Hoffman tree$\gamma_{ns2oe}(P_p{}^{+}) = p$

Proof:

Let V$(P_n{}^{+}) = \{v_1, v_2 v_3 - - - - - - v_p, v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$

Here $\{v_1, v_2 v_3 - - - - - - v_p\}$ be the vertices of the path, $\{ v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$ be the pendant edge attached at each vertex of the path.

Let D= $\{v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$ be minimal dominating set and $V - D$= $\{v_1, v_2 v_3 - - - - - - v_p\}$

Each vertices of path v_{pi} have a neighborhood in D and other $V - D$

$od_D(v_{pi}) = |N(v_{pi}) \cap V - D|$=1 for all i=1,2,3------n

$|od_D(v_{pi}) - od_D(v_{pj})| \leq 2$

Then D is two out degree equitable dominating set and<N(D)>= $V - D$ form a path , so

$< V - D >$connected

Then D is minimal non split two out degree equitable dominating set

Then $\gamma_{ns2oe}(P_p{}^{+}) = |D| = p$

Exact values of non-split two out degree equitable dominating for some special graphs

Peterson graph: The non-split two out degree equitable domination number of Peterson graph is 5

Proof:

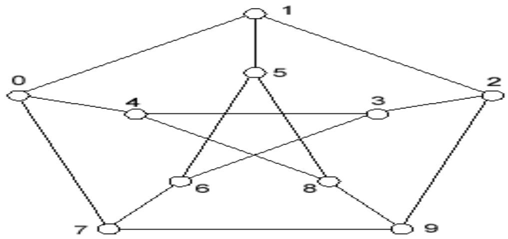

Figure.2.

Let us consider D={ 3,4,5,6,8} and $V - D$ = {0,1,2,7,9}

Clearly D is minimum two out degree equitable dominating set and the induced subgraph $< V - D >$ is connected

Diamond Graph: The diamond graph is a planer undirected graph with 4 vertices and 5 edges as show in figure 3 is consist of a complete graph K_4minus one edge

For any diamond graph G of order 4. $\gamma_{ns2oe}(W_4)$=2

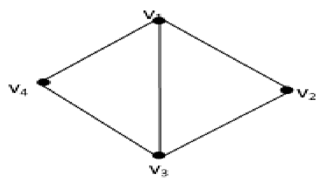

Figure.3

In the figure.D= $\{v_1, v_2\}$ is non spilt two out degree equitable dominating set

Fan graph: For any fan graph of order p≥ 4, $\gamma_{ns2oe}(F_{1,p-1})$= $\begin{cases} 2 & if\ p = 4,5 \\ p - 2 & if\ p \geq 6 \end{cases}$

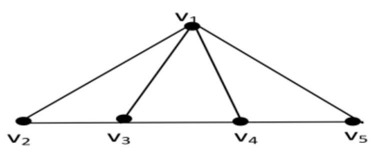

Fig.4(a)

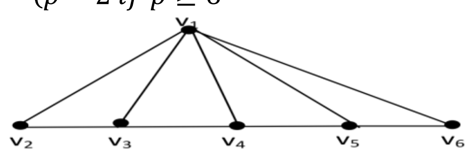

fig4.(b)

In figure 4(a) D= $\{v_1,v_2\}$ is non split two out degree equitable dominating set so $\gamma_{ns2oe}(F_{1,4}$)=2

In figure 4(b) D= $\{v_1, v_2, v_3, v_4\}$ is non-split two out degree equitable dominating set so $\gamma_{c2oe}(F_{1,5}$)=4

Moser spindle: The Moser spindle is an undirected graph with seven vertices and eleven edges as show in figure

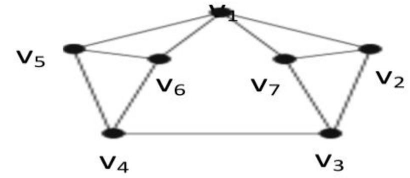

Figure.5

In above figure D= $\{v_1, v_{2,} v_3\}$ is a non-split two out degree equitable dominating set so $\gamma_{ns2oe}(G\ )$=3

Bull Graph: The Bull graph is a planner undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendent edges

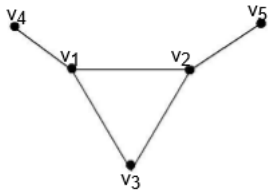

Figure.6

The non-split two out degree equitable domination number is 2.

Here $\{v_4, v_5,\}$ is a minimum non split two out degree equitable dominating set

Crown graph: Any cycle with a pendent edge attached at each vertex is shown in figure 2 is called Crown graph and is denoted by $C^+{}_p$

For the Crown graph, $\gamma_{ns2oe}(C^+{}_p\ )$=p

For figure 6 above theorem D= $\{v_1, v_2, v_3\}$ is minimum non split two out degree equitable dominating set

2. METHODS & MATERIALS

Here we collect some research papers from various journals and downloads some papers form the internet related to the domination in graph theory and we defined a new domination number and we study them in details.

3. RESULTS

In this paper a new domination number called non split two out equitable domination number is defined and this domination number for some standard and special graphs called Peterson graph, diamond graph, Fan graph, Moser spindle and Bull graph.

4. CONCLUSIONS

In this paper a new domination number called non split two out equitable domination number is defined and this domination number for some standard and special graphs. Further we are like to expend this research work for another group of graphs and we like to study the applications of the non-split two out degree equitable domination number.

REFERENCE

Berge C, Theory of graph and its applications, Methuen Londan, 1962.

Harray F, Graph theory, Addison-Wesly Reading MA, 1969.

Kulli VR and Janakiram B, The nonsplit domination number of a graph, Indian J Pure, Appl. Math, 31, 2000, 545-550.

Mahesh MS and Namasivayam P, Connected two out degree equitable domination number in different graphs, International Journal of Mathematical Archive, 5(1), 2014, 67-74.

Ore O, Theory of graphs, Amerarica, Math. Soc, Collog, Publications 38, Providence, 1962.

Sahal A and Mathad V, Two-out degree equitable Domination in graphs, Transactions on Combinatorics, 2(3), 2013, 13-19.