A Numerical Study of Double-Diffusive Magnetohydrodynamic Mixed Convection at a Vertical Plate

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ABSTRACT

A numerical study of double-diffusive magneto hydrodynamic (MHD) mixed convection at a permeable vertical plate is made in this article. The electrically conducting fluid is assumed to flow with a constant velocity parallel to the vertical plate. Viscosity of the fluid is taken to be a function of temperature. Using a similarity variable, governing equations of the two-dimensional boundary layer flow, heat and mass transfer are transformed to ordinary differential equations and solved by Nachtsheim-Swigert scheme. Assisting and opposing buoyancy cases are considered. The characteristics of flow, heat and mass transfer depend upon a number of non-dimensional parameters including Pr - the Prandtl number, Sc - the Schmidt number and θ_r - the viscosity variation parameter. The skin friction, the heat transfer coefficient and the Sherwood number are determined for a wide range of fluids ($0.7 \leq Pr \leq 250$), for different types of concentration species ($0.1 \leq Sc \leq 100$) and for different values of the viscosity variation parameter θ_r ($5 \leq \theta_r \leq 1000$). It is observed that: In assisting flow, skin friction reduces with increasing Prandtl number and diminishing diffusion of the species. It increases with increasing viscosity variation parameter. In opposing flow skin friction gets enhanced with diminishing diffusion of the species. In both assisting and opposing flows the Sherwood number increases, almost linearly, with diminishing diffusion of the species.

KEY WORDS: Mixed convection, Double diffusion, Magneto hydrodynamics.

1. INTRODUCTION

Double diffusive convection processes occur in many fields of Science and technology, for example in solid state physics, in Oceanography and in processes like drying, evaporation and condensation (Refer Pop and Ingham 2001). Many researchers have analyzed double-diffusive convection problems in free flows as well as in flow through porous media taking into consideration different aspects of the problems. Mahdy (2010), discussed double diffusive convection in porous media; (Subhashini, 2013; Teamah, 2013; Xu, 2014; Patil, 2014, 2016; Chen, 2004) are some of the many researchers who analyzed different double diffusive convection problems. The effects of magnetic field, chemical reaction, suction / injection etc. on double diffusive free convection and mixed convection were discussed by the earlier researchers. Gebhart and Pera (1971), presented similarity solutions of a double diffusive natural convection problem with water and air for many values of Schmidt number in aiding and opposing buoyancy cases. The effects of chemical reaction and variable viscosity on mixed convective heat and mass transfer at a permeable plate were discussed by Mahmoud (2007). Similarity solutions were presented by Rao (2016), for magneto hydrodynamic double diffusive free convection at a permeable vertical plate through the use of a buoyancy ratio parameter. Prasad (2016), made a similar analysis of magneto hydrodynamic double diffusive mixed convection at a permeable vertical plate and presented similarity solutions.

In this article the authors reanalyzed the problem of Prasad (2016), in a different approach, derived certain new results and presented them. In this article also, the governing equations are reduced to a set of ordinary differential equations by the use of a similarity variable and solved by Nachtsheim-Swigert scheme (Nachtsheim and Swigert, 1965). Assisting and opposing buoyancy cases are considered. Certain important results of the analysis showing the effects of different base fluids, different concentration species and viscosity variation on the characteristics of the flow, heat and mass transfer are presented and discussed.

Formulation & Solution: Consider a permeable plate immersed vertically in a homogeneous electrically conducting viscous fluid containing a concentration species. The fluid flows with a fixed velocity (u_∞) parallel to the plate. X-axis is chosen vertically upwards along the plate. Y-axis is chosen perpendicular to it. A magnetic field of intensity B_o acts transverse to the plate. Viscosity of the fluid is assumed to depend on temperature as $1/\mu=1/\mu_\infty [1+\alpha(T-T_\infty)]$. Using Boussinesque's approximation, introducing a similarity variable, the equations governing the two-dimensional laminar flow, heat and mass transfer can be written in non-dimensional form as

$$(\theta-\theta_r)f''' - f''\theta' - \frac{(\theta-\theta_r)^2}{2\theta_r} ff'' - \frac{(\theta-\theta_r)^2}{\theta_r}\frac{Gr_x}{{Re_x}^2}\,\theta - \frac{(\theta-\theta_r)^2}{\theta_r}\frac{Grc_x}{{Re_x}^2}\,\emptyset + \frac{(\theta-\theta_r)^2}{\theta_r}\frac{M_x^2}{Re_x} f' = 0 \qquad (1)$$

$$\theta'' + \frac{1}{2}\,Pr\,f\,\theta' + \frac{M_x^2\,Pr\,Ec}{Re_x}\,f'^2 = 0 \qquad (2)$$

$$\emptyset'' + \frac{1}{2}\,Sc\;f\emptyset' - \gamma_x Sc Re_x \emptyset = 0 \qquad (3)$$

The boundary conditions for f, θ, ∅ are

$$\left.\begin{aligned} &at\,\eta = 0,\ f = v_w,\ f' = 0, \theta = 1, \varnothing = 1 \\ &as\ \eta \to \infty,\ f' \to 1, \theta \to 0,\ \varnothing \to 0 \end{aligned}\right\} \qquad (4)$$

In the above equations and boundary conditions, η (similarity variable), f (non-dimensional stream function), θ (non-dimensional temperature), ∅ (non-dimensional concentration), θ_r (viscosity variation parameter), Gr_x (local Grashof Number for the fluid), Re_x (local Reynolds number), Grc_x (Grashof Number for the concentration species), M_x (Magnetic Reynolds number), Pr (Prandtl number), Ec (Eckert number (due to Ohmic heating)), Sc (Schmidt number), γ_x (chemical reaction parameter) and v_w (suction / injection parameter) have the same definitions of Prasad (2016), and they are not presented here for sake of brevity. Numerical solutions of the equations (1) – (3) with boundary conditions (4) are obtained by the use of Nachtsheim-Swigert scheme for certain physically meaningful as well as certain hypothetical values of parameters. For certain selected values of parameters the results are compared with those of Prasad (2016), and found good agreement between the two.

3. RESULTS AND DISCUSSION

The flow, heat and mass transfer characteristics of practical interest are the skin friction (shear stress at the plate), Nusselt number (heat transfer rate at the plate) and Sherwood number (mass transfer rate at the plate) besides the hydrodynamic, thermal and concentration boundary layer thicknesses. In this paper, attention is confined to the behaviors of skin friction, Nusselt number and Sherwood number for values of the parameters, $0.7 \leq Pr \leq 250$, $0.1 \leq Sc \leq 100$, $5 \leq \theta_r \leq 500$ and for $M_x = 0.0, 0.1$, $Ec = 0.0, 0.1$, $v_w = 0.25, 0.5$, $Gr_x = 5$, $Grc_x = -2, 4, -4$, $\gamma_x = 0.1, 0.2$, $Re_x = 3.0$. When the buoyancy forces due to the fluid and the species act in the same direction or in opposite directions, the resulting flow will be referred to as assisting flow or opposing flow. In this analysis, negative values of Grc_x correspond to opposing flow. Also note that 'Pr = 0.7' corresponds to the base fluid air while 'Pr = 250' corresponds to unused engine oil at about 110^0C. The variations in viscosity for air are not at all significant while they are relatively significant for unused engine oil. (In the figures 'Gm' is used to represent γ_x.)

Variations in skin friction ($f''(0)$) with the Schmidt number Sc (in fact ln Sc – Logarithm of Sc) in assisting flow are shown in figure -1. Similar variations of Nusselt number($-\theta'(0)$) and Sherwood number($-\emptyset'(0)$) are shown in figures.2, 3. From figure.1, one may note that skin friction diminishes with diminishing diffusion of the species (increasing Sc) and tend to approach a constant value as the diffusion further diminishes. Further, increasing chemical reaction and increasing Prandtl number diminish skin friction. From figure.2, Nusselt number can be seen to be unaffected by changing values of the Schmidt number Sc. However, increasing Prandtl number significantly increases Nusselt number while chemical reaction parameter slightly diminishes Nusselt number. From figure.3, the Sherwood number can be seen to increase exponentially with 'ln Sc', or Sherwood number grows almost linearly with Sc. Increasing chemical reaction increases Sherwood number and Prandtl number has insignificant effect on Sherwood number.

In figures.4 to 6 are shown variations in $f''(0)$, '$-\theta'(0)$', '$-\emptyset'(0)$' with 'ln Sc' in opposing flow. Contrary to assisting flow, in opposing flow, skin friction grows with diminishing diffusion of the species and approaches a constant value as the diffusion further diminishes. Also increasing Prandtl number has a diminishing effect on skin friction. From figures.5 and 6, one can note that the variations in '$-\theta'(0)$', $-\emptyset'(0)$' in opposing flow are exactly similar to those of the assisting flow.

In figures.7 to 9, are presented plots of $f''(0)$, '$-\theta'(0)$', '$-\emptyset'(0)$' against the viscosity variation parameter θ_r (in fact '$ln\ \theta_r$') in opposing flow for Pr = 250. As mentioned earlier, variation of viscosity with temperature for the fluid represented by Pr = 250, unused engine oil, is relatively significant. From figure.7, it may be noted that skin friction grows with increasing θ_r. That is, if viscosity variation with temperature is neglected, then skin friction can assume relatively larger values. Further, when there is no magnetic field (and hence no Ohmic heating), skin friction assumes relatively larger values. From figures 8 and 9 one can note that the behaviours of '$-\theta'(0)$' and '$-\phi'(0)$' are qualitatively similar to those of '$f''(0)$'.

Figures.10 to 12 depict behaviours of $f''(0)$, '$-\theta'(0)$', '$-\phi'(0)$' with changing Prandtl number Pr in assisting flow. From figure-10, skin friction can be seen to diminish with increasing Pr. The variation in skin friction between suction and injection cases reverses with increasing Prandtl number. In the presence of suction Nusselt number increases with Prandtl number while in the presence of injection, opposite is the behaviour (figure.11). Also the variations in the Nusselt number with Pr are significant in the suction case while they are insignificant in the injection case. Further, increasing Prandtl number diminishes Sherwood number although the diminishing nature is not at all significant (figure.12).

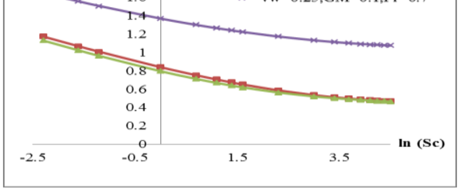

Figure.1. Plots of skin friction for $Mx = Ec = 0.1, Re_x = 3, Gr_x = 5, Grc_x = 4, \Theta_r = 500$

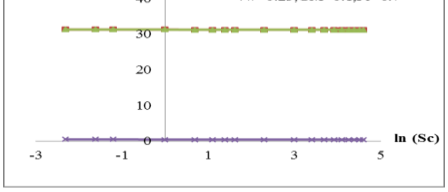

Figure.2. Plots of Nusselt number for $Mx = Ec = 0.1, Re_x = 3, Gr_x = 5, Grc_x = 4, \Theta_r = 500$

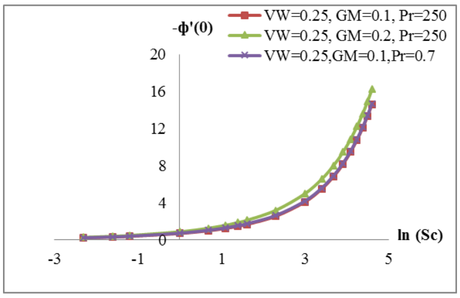

Figure.3. Plots of Sherwood number for $Mx = Ec = 0.1, Re_x = 3, Gr_x = 5, Grc_x = 4, \Theta_r = 500$

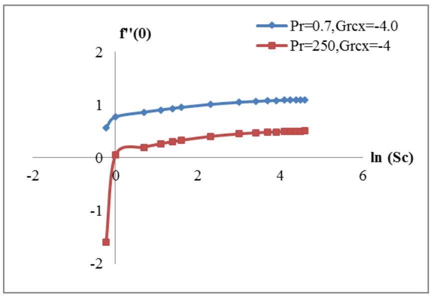

Figure.4. Plots of skin friction for $Mx = Ec = Gm = 0.1, Vw = 0.5, Sc = 0.6, Gr_x = 5, \Theta_r = 500$

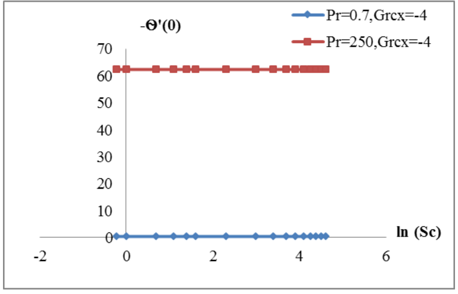

Figure.5. Plots of Nusselt number for $Mx = Ec = Gm = 0.1, Vw = 0.5, Sc = 0.6, Gr_x = 5, \Theta_r = 500$

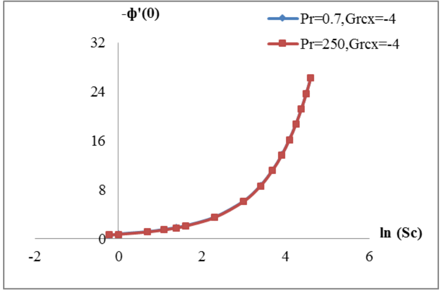

Figure.6. Plots of Sherwood number for $Mx = Ec = Gm = 0.1, Vw = 0.5, Sc = 0.6, Gr_x = 5, \Theta_r = 500$

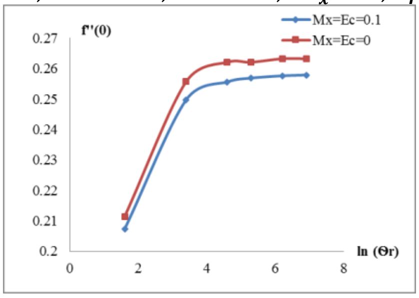

Figure.7. Plots of skin friction for $Gm = 0.1, Vw = 0.25, Sc = 2.0, Pr = 250, Gr_x = 5, Grc_x = -2, Re_x = 3$

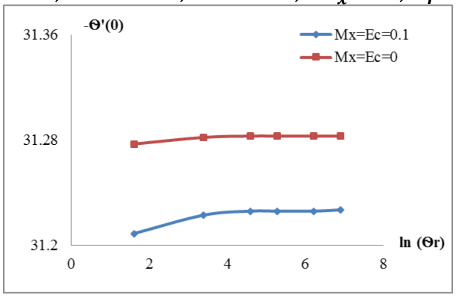

Figure.8. Plots of Nusselt number for $Gm = 0.1, Vw = 0.25, Sc = 2.0, Pr = 250, Gr_x = 5, Grc_x = -2, Re_x = 3$

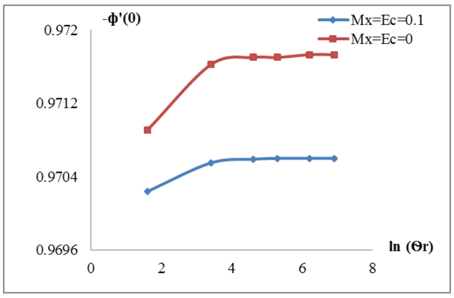

Figure.9. Plots of Sherwood number for $Gm = 0.1, Vw = 0.25, Sc = 2.0, Pr = 250, Gr_x = 5, Grc_x = -2, Re_x = 3$

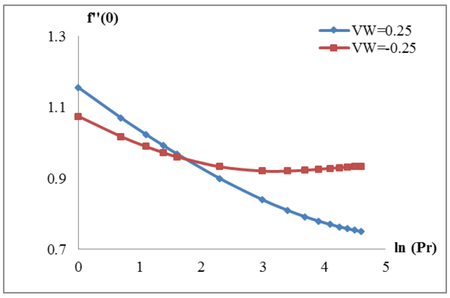

Figure.10. Plots of skin friction for $Mx = Ec = Gm = 0.1, Sc = 0.6, Gr_x = 5, Grc_x = 4, Re_x = 3, \Theta_r = 5$

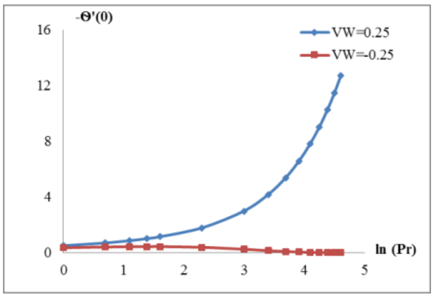

Figure.11. Plots of Nusselt number for $Mx = Ec = Gm = 0.1, Sc = 0.6, Gr_x = 5, Grc_x = 4, Re_x = 3, \Theta_r = 5$

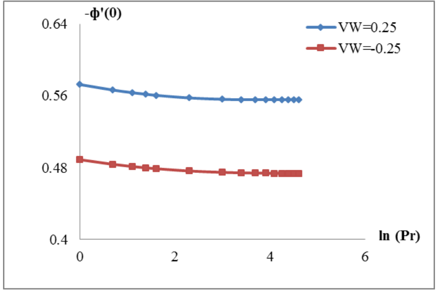

Figure.12. Plots of Sherwood number for $Mx = Ec = Gm = 0.1, Sc = 0.6, Gr_x = 5, Grc_x = 4, Re_x = 3, \Theta_r = 5$

4. CONCLUSION

- With aiding buoyancies, as the rate of diffusion diminishes skin friction diminishes and asymptotically approach a constant value. Increasing chemical reaction and increasing Prandtl number also diminish skin friction.
- With opposing buoyancies skin friction grows with diminishing diffusion of the species and asymptotically approach a constant value. Also skin friction grows with the Prandtl number.
- With both aiding and opposing buoyancies Sherwood number increases almost linearly with increasing Sc or with diminishing diffusion of the species. Nusselt number is not significantly affected by the Schmidt number as well as viscosity variation parameter.

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REFERENCES

Chen C.H, Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation, International journal of engineering science, 42(7), 2004, 699-713.

Gebhart B and Pera L, The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, International Journal of Heat and Mass Transfer, 14(12), 1971, 2025-2050.

Mahdy A, Chamkha A.J and Baba Y, Double-diffusive convection with variable viscosity from a vertical truncated cone in porous media in the presence of magnetic field and radiation effects, Computers & Mathematics with Applications, 59 (12), 2010, 3867-3878.

Mahdy A, Effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity, International Communications in Heat and Mass Transfer, 37 (5), 2010, 548-554.

Mahmoud M.A, A note on variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a semi-infinite vertical plate, Mathematical Problems in Engineering, 2007, 1-7.

Nachtsheim P.R and Swigert P, Satisfaction of Asymptotic Boundary Conditions in Numerical solution of systems of nonlinear equations of Boundary-layer type, NASA TN D-3004, Washington D.C, 1965.

Patil P.M, Momoniat E and Roy S, Influence of convective boundary condition on double diffusive mixed convection from a permeable vertical surface, International Journal of Heat and Mass Transfer, 70, 2014, 313-321.

Patil P.M, Roy S and Momoniat E, Thermal diffusion and diffusion-thermo effects on mixed convection from an exponentially impermeable stretching surface, International Journal of Heat and Mass Transfer, 100, 2016, 482-489.

Pop I and Ingham D.B, Convective heat transfer: mathematical and computational modelling of viscous fluids and porous media, Elsevier, 2001.

Prasad T.R.K.D.V, Rao C.N.B and Prasad P.H, Double Diffusive Mixed Convection at a Vertical Plate in the Presence of Magnetic Field, International Journal of Chemical Sciences, 14 (2), 2016, 923-935.

Rao S.R, Rao C.N.B and Chandra Shekar K.V, Double Diffusive Magneto hydrodynamic Free Convection at a Vertical Plate, International Journal of Chemical Sciences, 14 (2), 2016, 978-992.

Subhashini S.V, Sumathi R and Pop I, Dual solutions in a double-diffusive MHD mixed convection flow adjacent to a vertical plate with prescribed surface temperature, International Journal of Heat and Mass Transfer, 56 (1), 2013, 724-731.

Teamah M.A, Sorour M.M, El-Maghlany W.M and Afifi A, Numerical simulation of double diffusive laminar mixed convection in shallow inclined cavities with moving lid, Alexandria Engineering Journal, 52 (3), 2013, 227-239.

Xu H, Xiao R, Karimi F, Yang M and Zhang Y, Numerical study of double diffusive mixed convection around a heated cylinder in an enclosure, International Journal of Thermal Sciences, 78, 2014, 169-181.