Combined Solutal and Thermal Buoyancy in Magneto hydrodynamic Free Convection at a Vertical Plate - A Numerical Study

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ABSTRACT

In this article, double diffusive free convection at a permeable vertical plate in the presence of a transverse magnetic field and chemical reaction is analysed numerically. Viscosity is assumed to be temperature dependent. A similarity variable is introduced. Equations governing the boundary layer flow, heat and mass transfer are transformed into a set of non-linear ordinary differential equations. Nachtsheim-Swigert scheme is used for obtaining the solutions. Assisting and opposing flow cases are considered. Characteristics related to the flow, heat and mass transfer are determined for a range of values of the Prandtl number Pr $(0.7 \leq Pr \leq 250)$ that correspond to different fluids; for a number of values of the Schmidt number Sc $(0.1 \leq Sc \leq 100)$ that correspond to different concentration species and for a number of values of the Grashof number Gr_x $(0.05 \leq Gr_x \leq 100)$ and for certain values of other parameters. Some important observations of the analysis are: In assisting flow as well as in opposing flow skin friction diminishes as Prandtl number increases. In opposing flow, skin friction assumes relatively smaller numerical values and also assume negative values for $Sc \leq 30$. The heat transfer coefficient increases, almost linearly with increasing values of Pr. Like skin friction, the Sherwood number also diminishes with increasing values of Pr. Skin friction gets diminished in assisting flow and gets enhanced in opposing flow with diminishing diffusion of the species.

KEY WORDS: Free convection, Solutal and Thermal Buoyancy (Double diffusion), Magneto hydrodynamics.

1. INTRODUCTION

Double diffusive processes occur in different fields of Science and Technology including Solid state physics, Oceanography and in processes like drying, evaporation, sublimation etc.(Pop and Ingham, 2001). There is a plethora of papers in literature dealing with double diffusion in flow through porous media and in free flows. (Prasad and Hemalatha, 2016; Rashidi, 2014; Hirschhorn, 2016; Anwar, 2012; Ravikumar, 2013) are among many researchers who analyzed double diffusive convection processes under different constraints. Effects of magnetic field, permeability of the wall, chemical reaction, porous media, heat sources and variable fluid properties are among some of the aspects discussed by the authors of earlier works. Mahmoud (2007), used similarity solutions to study the effects of variable viscosity and chemical reaction on mixed convective heat and mass transfer at a porous vertical plate. In the presence of internal heat generation and chemical reaction, Patil (2010), analyzed steady double diffusive mixed convection at a moving vertical plate by a non-similarity analysis. Mixed convective heat and mass transfer at a vertical plate was analysed by Geetha and Moorthy (2011), to study the effects of chemical reaction, variable viscosity and thermal stratification. Prasad (2016), studied double diffusive mixed convection at a vertical plate through the use of a buoyancy ratio parameter. A similar double diffusive free convection analysis was made by Rao (2016).

In the present paper the authors reanalyzed the problem of Rao (2016), in a different manner, in the cases of assisting and opposing buoyancies. The variations in the skin friction, the Nusselt number and the Sherwood number are emphasized for different base fluids and different concentration species when viscosity of the fluid varies as an inverse linear function of temperature.

Formulation and Solution: A magnetic field of intensity B_0 is applied transverse to a permeable plate immersed vertically in a viscous, homogeneous, electrically conducting fluid containing a concentration species. $X-$axis is taken vertically upwards along the plate. $Y-$ axis is taken perpendicular to it. Viscosity of the fluid is assumed to vary as a function of temperature as $\frac{1}{\mu} = \frac{1}{\mu_\infty}[1+\propto (T - T_\infty)]$.

Using Boussinesque approximation and a similarity variable, the equations governing the two-dimensional laminar boundary layer flow, heat and mass transfer are presented in non- dimensional form as

$$(\theta-\theta_r)f''' - f''\theta' - 3ff''\frac{(\theta-\theta_r)^2}{\theta_r} + 2f'^2\frac{(\theta-\theta_r)^2}{\theta_r} - \frac{(\theta-\theta_r)^2\theta}{\theta_r} - \frac{Grc_x}{Gr_x}\frac{(\theta-\theta_r)^2}{\theta_r}\emptyset + \frac{2M_x{}^2}{\sqrt{Gr_x}}\frac{(\theta-\theta_r)^2}{\theta_r}f' = 0 \quad (1)$$

$$\theta'' + 3Prf\theta' + 8M_x{}^2\lambda_x f'^2 = 0 \quad (2)$$

$$\phi'' + 3Scf\phi' - 2\gamma_x Sc\phi = 0 \quad (3)$$

The boundary conditions in terms of f, θ, ϕ are

$$at\ \eta = 0, \quad f = v_w,\ f' = 0,\ \theta = 1,\ \phi = 1$$
$$as\ \eta \to \infty, \quad f' = 0,\ \theta = 0,\ \phi = 0 \quad (4)$$

In these equations and boundary conditions, η (similarity variable), θ (non dimensional temperature), f(non dimensional stream function), ϕ (non dimensional concentration), as well as the parametersθ_r (viscosity variation parameter), v_w (suction/injection parameter), Gr_x(local Grashof Number for the fluid), Grc_x (Grashof Number for the concentration species), M_x (Magnetic Reynolds number), Pr (Prandtl number), γ_x(chemical reaction parameter) and Sc (Schmidt number) are as defined in Rao (2016) and are not given here. Further, $\lambda_x = Pr\frac{\beta g x}{c_p}\frac{1}{\sqrt{Gr_x}}$ is a parameter describing the effect of Ohmic heating.

The asymptotic boundary value problem consisting of equations (1) - (3) and conditions (4) is solved by Nachtsheim - Swigert technique (Nachtsheim and Swigert, 1965) in assisting and opposing flow cases for certain practical as well as fictitious values of the parameters of the problem. To test the validity of the results of the analysis, some results, for particular values of parameters are compared with those of Rao (2016).

3. DISCUSSION OF THE RESULTS

Some of the qualitatively and quantitatively distinct results related to the skin friction ($f''(0)$, the Shear stress at the plate), the Nusselt number $(-\theta'(0)$, the heat transfer rate) and the Sherwood number $(-\emptyset'(0)$, the mass transfer rate) corresponding to the values of the parameters $0.7 \le Pr \le 250$, $0.1 \le Sc \le 100$, $0.05 \le Gr_x \le 100$, θ_r= 5, 500, $M_x = 0, 0.2$, $\lambda_x = 0, 0.2$, $v_w = 0.2$, $\gamma_x = 0.2$, $Grc_x = 4, -2$ are presented in the form of graphs and discussed. It may be noted that the fluid and species buoyancies may act in the same direction or in opposite directions and accordingly, there can be two resulting flows assisting flow and opposing flow. In the present analysis, a positive value for Grc_x correspond to assisting flow while a negative value corresponds to opposing flow. In the figures Gmx is used to represent γ_x.

In figures 1 to 3 are presented plots of the skin friction($f''(0)$) , the Nusselt number $(-\theta'(0))$ and the Sherwood number$(-\emptyset'(0))$ versus the Schmidt number Sc, or infact versus logSc (logarithm of Sc) in assisting flow for $Pr = 250$. As the diffusion of the species decreases (or as Sc increases), skin friction diminishes and tend to approach a constant value (refer to fig.1). For the fluid corresponding to $Pr = 250$ i.e., 'unused engine oil around $110^0 C$', the rate of change of viscosity with temperature is relatively high and so θ_r can assume a numerical value of order unity. Numerical values of $f''(0)$ corresponding to θ_r=5 are small compared to those for θ_r=500 indicating that if we ignore variation of viscosity with temperature, $f''(0)$ can assume relatively larger values.

Increasing values of Sc cause an increase in the Nusselt number while further increase has no effect of the Nusselt number (refer to fig. 2). The Sherwood number increases gradually with diminishing diffusion of the species (up to $Sc = 30$), and increases almost linearly for further diminishing of the diffusion (fig.3). The viscosity variation parameter has no significant effect on the Nusselt number and Sherwood number.

Figures 4 to 6 depict changes in $f''(0)$,'$-\theta'(0)$', '$-\emptyset'(0)$' versus 'logSc' in opposing flow for $Pr = 93$. Unlike in assisting flow, in this case, $f''(0)$ takes negative values for a range of values of Sc, and become positive beyond a certain stage. Also, in opposing flow, $f''(0)$increases with increasing values of Sc (refer to fig 4). Skin friction for θ_r= 5 assumes relatively larger numerical values than those for θ_r= 500. Further, the viscosity variation for the fluid corresponding to $Pr = 93$, namely Ethylene Glycol (at around $40^0 C$) is relatively high and an appropriate value of θ_r for this fluid can be θ_r= 5. The Nusselt number increases with diminishing diffusion of the species as well as the viscosity variation parameter θ_r (fig.5). From figure.6, the Sherwood number can be seen to vary almost exponentially with '$logSc$' indicating that the Sherwood number increases linearly with Sc. Further θ_r has negligible effect on the Sherwood number.

Figures.7 to 9 show changes in $f''(0)$,'$-\theta'(0)$', '$-\emptyset'(0)$' with the Prandtl number Pr, or, infact with '$logPr$' in an assisting flow. Skin friction can be seen to be a diminishing function of the Prandtl number and an increasing function of θ_r (fig.7). The Nusselt number varies as an exponential function of '$logPr$', or as a linear function of Pr and is not significantly effected by changing values of θ_r(fig.8). From figure.9, the Sherwood number, like the skin friction, can be seen to diminish with increasing Prandtl number and increase with increasing values of θ_r.

In figures.10, 11, 12 are presented variations of $f''(0)$,'$-\theta'(0)$', '$-\emptyset'(0)$' with changing Grashof number Gr_x, or, with changing '$logGr_x$' in an assisting flow. Skin friction diminishes with increasing Gr_x and asymptotically approach zero. Numerical values of $f''(0)$ are larger for suction at the plate and smaller for injection (refer to fig.10). Nusselt number, like skin friction is a diminishing function of Gr_x and assumes larger values for suction than for injection (fig.11). The behavior of Sherwood number is exactly similar to that of the Nusselt number for both suction and injection (fig.12).

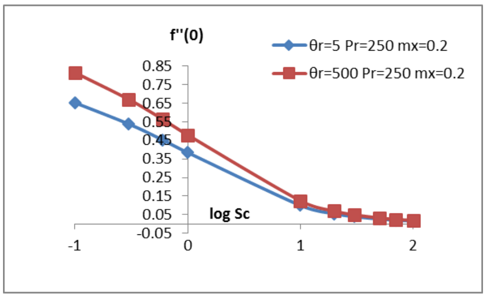

Figure.1. Plots of skin friction for λ=0.2, Vw=0.2,Grx=5,Grcx=4,Gmx=0.2

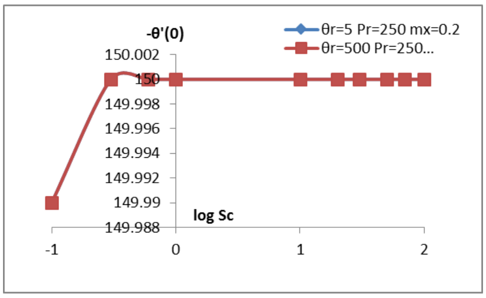

Figure.2. Plots of Nusselt number for λ=0.2 ,Vw=0.2,Grx=5,Grcx=4,Gmx=0.2

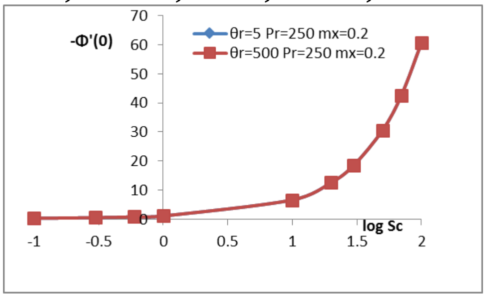

Figure.3. Plots of Sherwood number for λ=0.2, Vw=0.2,Grx=5,Grcx=4 ,Gmx =0.2

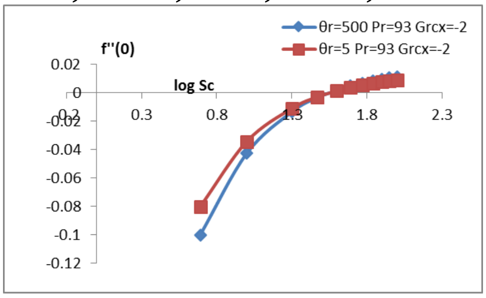

Figure.4. Plots of skin friction for mx=0.2,λ=0.2, Vw=0.2,Grx=5,Gmx=0.2

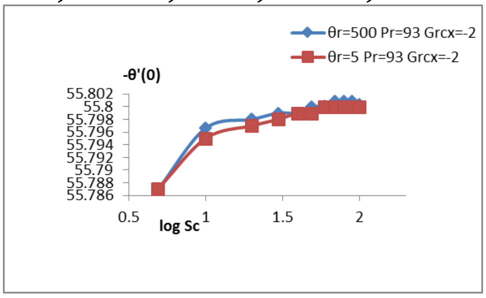

Figure.5. Plots of Nusselt number for mx=0.2,λ=0.2,Vw=0.2,Grx=5,Gmx=0.2

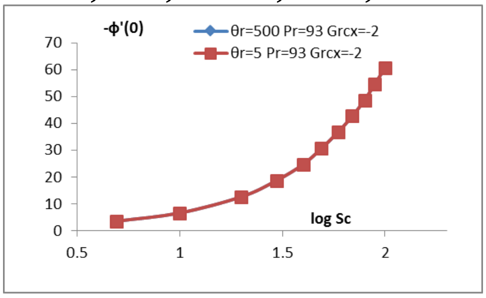

Figure.6. Plots of Sherwood number for mx=0.2,λ=0.2,Vw=0.2, Grx=5,Gmx=0.2

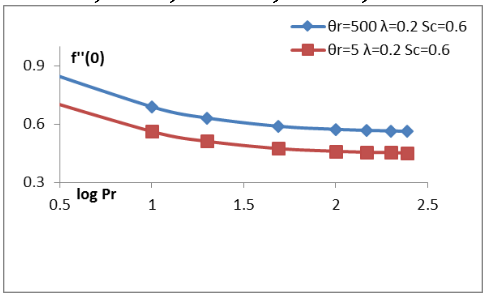

Figure.7. Plots of skin frction for Vw=0.2,mx=0.2,Grx=5,Grcx=4,Gmx=0.2

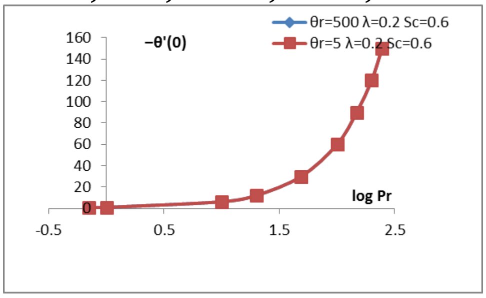

Figure.8. Plots of Nusselt number for Vw=0.2,mx=0.2,Grx=5,Grcx=4,Gmx=0.2

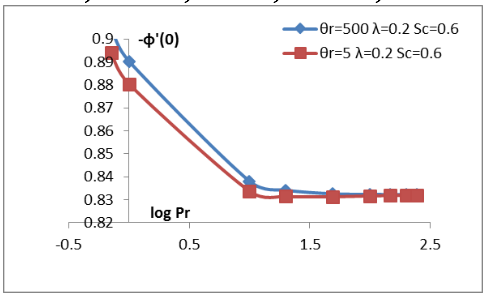

Figure.9. Plots of Sherwood number for Vw=0.2, mx=0.2,Grx=5,Grcx=4,Gmx=0.2

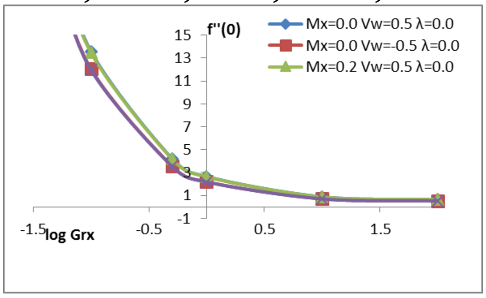

Figure.10. Plots of skin friction for Pr=0.7,Grcx=5,θr=500,Gmx=0.2,Sc=0.6

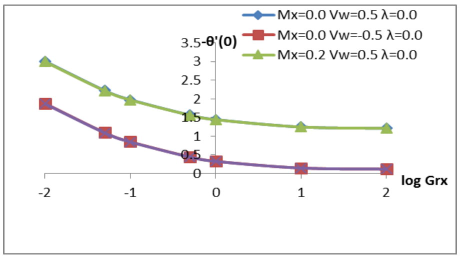

Figure.11. Plots of Nusselt number for Pr=0.7,Grcx=5,θr=500,Gmx=0.2,Sc=0.6

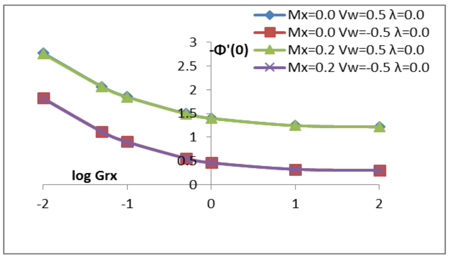

Figure.12. Plots of Sherwood number for Pr=0.7,Grcx=5,θr=500,Gmx=0.2,Sc=0.6

4. CONCLUSION

In assisting flow Skin friction diminishes and Sherwood number increases with diminishing diffusion of the species (or with increasing values of Sc). Skin friction and Sherwood number are decreasing functions of the Prandtl number and increasing functions of θ_r.

In opposing flow, Skin friction becomes negative for relatively high diffusing species, increases, and becomes positive for diminishing diffusion of the species. In opposing flow also Sherwood number increases with diminishing diffusion of the species.

In assisting flow as well as in opposing flow Nusselt number increases with diminishing diffusion of the species. Also Nusselt number increases with the Prandtl number.

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