Comparison of eigen frequency convergence criteria on a bridge structure

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ABSTRACT

In this paper the eigen frequency predictions were compared on a common Bridge truss model for the effectiveness of the four different eigen frequency calculations techniques. The matrix formulation was revisited and different aspects of matrix formulations were used. Both the distributed mass matrix and lumped mass matrix formulation were considered. The above combinations were repeated with simple matrix multiplication. A MATLAB based code was developed for eigen frequencies computation with different combinations but on a common bridge truss structure model. The results were compared for the effectiveness of the algorithms on a common structure for their effectiveness of the convergence criteria. The graphs were plotted for various cases. The results are discussed at the end.

KEY WORDS: System Identification, Finite Element, Structure dynamics, mode shape, mass matrix, Eigen values, Eigen vectors.

1. INTRODUCTION

It becomes important to understand the dynamic behaviour of a structure, especially for structures which are strongly dependent on the structural dynamic characteristics for their operational performance. Determination of natural frequencies is also an important parameter especially when the dynamic analysis of a multiple-degrees-of-freedom (MDOF) system is to be carried out. The dynamic response of a structural system is primarily governed by the natural frequencies and mode shapes Natasa Trisovic (2007).

Linear and non-linear problems with dynamic analysis of eigen values and eigen vectors plays very important role in the accurate prediction technique. Also, the techniques and the variations used in the field of system identification are growing at a very fast rate. With this rapid development in the field of system identification it becomes necessary to check the convergence effectiveness of these techniques on eigen frequency analysis.

Eigen frequency evaluation is usually required to obtain the mode shape and the natural frequency of the structure. The Eigen value is calculated from the stiffness and mass matrix of a structure Papadopoulos (1997). These parameters are employed for the evaluation of the Eigen frequency.

Many researchers used different techniques to find the eigen values & the eigen vectors. The technique which is to be adopted for solving a given problem depends upon the characteristic of the problem and the type of solution required. The characteristics mainly depend upon the size of the problem and the type of solution required depends upon the, range of required eigen values and eigen vectors.

The solution technique adopted by the commercial software's such as SAP, ABAQUS and ANSYS for evaluating the Eigen problem is the subspace iteration technique such as Householder-QR- inverse iteration method by In-Won Lee (1999).

The sub space iteration method, the inverse iteration method, simultaneous iteration method and Rayleigh-Ritz analysis by were approached by many researches. The subspace iteration method was developed independently by Dong (1972), Bathe (1971), and Bathe and Wilson (1973). It has been widely used and has proved to be efficient for solving the eigen problem of structures with a large bandwidth, Wilson and Itoh (1983).

The mass matrix formulation for eigen values can be calculated namely by two methods using lumped mass matrix or by using distributed by Chih-Peng-Yu and Jose M Roesset (2001) states that the analytical determination of dynamic stiffness matrices in the frequency domain for linear structural members with distributed mass provides an efficient and accurate procedure for the dynamic analysis of frames.

The eigen frequencies calculated by the lumped mass & the distributed mass matrix gives the different values. Thus, there is a need to analyse the structure by using both the methods for evaluating the eigen frequencies.

In this paper the eigen value predictions were compared on a common Bridge truss model for the effectiveness of the four different eigen frequency calculations techniques. The matrix formulation was revisited and different aspects of matrix formulations were used. Both the distributed mass matrix and lumped mass matrix formulation were considered. The above combination was repeated with the simple matrix multiplication for the calculations of eigen frequencies. The eigen frequencies were computed with these different combinations on the common bridge truss structure model. The results were compared on their effectiveness for their convergence criteria.

Eigen Frequency Prediction - Theoretical Development: In this section a breif introduction of the different methods, namely the Vianello Stodalla method, inverse power method, Rayleigh quotient iteration method and modified power method are stated for eigen frequency evaluation.

Vianello Stodalla method (Power method): With reference from Izadi F (2014), if λ is an eigen value of A that is larger in absolute value than any other eigen value, it is called the dominant eigen value. An eigenvector v corresponding to λ is called a dominant eigen vector. We assumed that the n×n matrix A has n distinct eigen values

λ_1, λ_2..... λ_n with corresponding eigenvectors of V_1, V_2,...... V_n and that eigen values are ordered in decreasing magnitude; that is, $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \ldots \geq |\lambda_n|$
The algorithm of the power method goes like this:
Pick a starting vector $x^{(0)}$ with $\|x^{(0)}\| = 1$

1. $w^{(0)} = Ax^{(0)}$ [1]
2. $C_1 = w_j^{(0)}$ in which $w_j^{(0)} = \|w^0\|_\infty$ [2]
3. $x^{(1)} = \frac{1}{C^1}x^0$ and $w^{(1)} = Ax^{(1)}$
4. While $|C_{k+1} - C_k| > eps$
   a. $C_{k+1} = w_j^{(k)}$ in which $w_j^{(k)} = \|w^k\|_\infty$ [3]
   b. $x^{(k+1)} = \frac{1}{C_{k+1}}x^{(0)}$ and $w^{(k+1)} = Ax^{(k+1)}$ [4]

Then the sequences $\{x^{(k)}\}$ and $\{C_k\}$ generated recursively converge to v_1 and λ_1.
Inverse power method: was referred by G.H. Golub and C.F. Van Loan (1996). The method gives. A technique widely used to obtain the smallest eigen value of a positive semi-definite symmetric matrix by Golub and Van Loan (1996)

We know that if λ is an eigen value of A and if A is non-singular then λ^{-1} is an eigen value of A^{-1}.
This suggests a way to estimate the smallest eigen value of A using the power method: arrange eigen values as
$|\lambda_1| \geq |\lambda_2| \geq \cdots \leq |\lambda_{n-1}| > |\lambda_n| > 0$ [5]
Since, A is non-singular, 0 is not an eigen value. The eigen values of A^{-1} arranged:
$|\lambda_n^{-1}| \geq |\lambda_{n-1}^{-1}| \geq \cdots \geq |\lambda_1^{-1}| > 0$ [6]
Therefore, apply power method to A^{-1}. But we don't compute A^{-1}. Instead solve
$Ax^{(k+1)} = x^{(k)}$ [7]
For $x^{(k+1)}$ by some efficient linear algebra solver. These two suggest a way to find the eigen value farthest to a given value μ. The "Shifted Matrix Power Method", here μ is complex generally: the trick is to construct a matrix.
$\hat{A} = (A - \mu I)$ [8]
And then use the regular power method on Â, i.e.
$x^{(k+1)} = \hat{A}x^{(k)}$ [9]
Finally, we could consider the eigen value closest to μ. In this case we apply the inverse power method on Â, i.e. $Ax^{(k+1)} = x^{(k)}$ [10]
Rayleigh Quotient iteration method: is an eigen value algorithm which extends the idea of the inverse iteration by using the Rayleigh Quotient to obtain increasingly accurate eigen value. A rapid convergence is guaranteed and no more than a few iterations are needed in practise. The process begin by choosing some value μ_0 as an initial eigen value guess for the matrix A. An initial vector b_0 must also be supplied as initial eigen vector guess. Calculate the next approximation of the eigen vector b_{i+1} by
$b_{i+1} = \frac{(A-\mu_i I)^{-1}b_i}{\|(A-\mu_i I)^{-1}b_i\|}$ [11]
Where I is the identity matrix, and
$\mu_i = \frac{b_i^* A b_i}{b_i^* b_i}$ [12]
Modified power method: by Palej Rafal (2015) presents a new approach to the power method serving the purpose of solving the eigen value problem of a matrix. Instead of calculating the eigen vector corresponding to the dominant eigen value from the formula
$V_{j+i} = AV_i$ [13]
The matrix B associated with matrix A is calculated from the formula
$B_{i+1} = k_i^m B_i^m$ [14]
Where, B_1 = A and m stands for the method's rate of convergence.

The computations are done by developing the algorithms of the above mentioned techniques, and running them using the MATLAB software. Once, the Eigen frequencies are calculated suitable graph has been plotted and inference has been drawn.

2. METHODOLOGY

Eigen Frequency Comparison on a common Bridge Truss Structure Model: A bridge truss structure model is taken from Agrawal, (1990). It is assumed that all elements are having the same modulus of elasticity E = 210GPa along with initial undamaged cross sectional area A=1.61×10^5 mm^2. The stiffness and mass matrix of the structure is calculated using the Finite element technique. The Bridge Truss[1], Fig. 1 is used for mass matrix formulation as a common structure for all eigen computations in this paper. Finite element technique is used for the stiffness and the mass matrix formulation for bridge truss. The lumped mass matrix and the distributed mass matrix formulations have been used for computation. On computation part of matrix multiplication the simple matrix multiplication as has been used. The eigen frequencies are computed with these different combinations. The results are compared on their effectiveness for their convergence criteria. The graphs have been plotted for various cases. The loads have been as in according to IRC: 6-2014.

Using the Vianello Stodalla: Vianello Stodalla method for Eigen frequency comparison was conducted by taking the different combinations of lumped mass matrix and distributed mass matrix along with the simple matrix multiplication. A MATLAB program code was used for computations & its plots. The results of eigen frequency are plotted by different combinations are shown in Fig. 2-3.

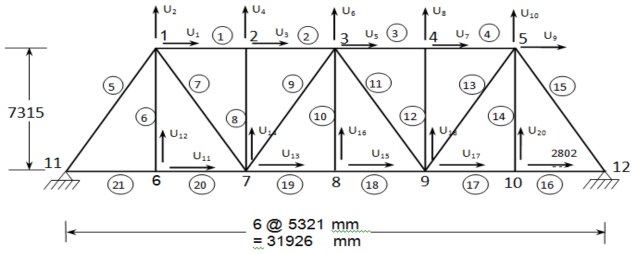

Figure.1. Railway bridge truss

Fig.2 is showing the program out come with lumped mass parameters in mass matrix formulation along with simple matrix multiplication in matrix multiplication.

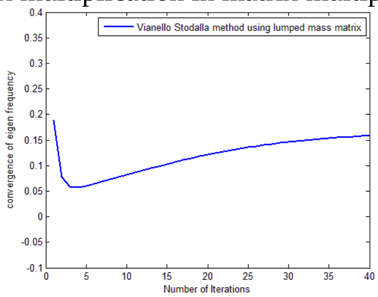

Figure.2. (a) is showing the Convergence of eigen frequency

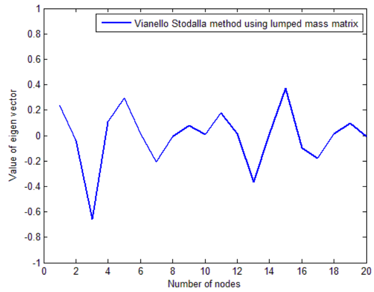

Figure.2. (b) showing the eigen frequency plot for each node at converged value.

Fig.3 is showing the program out come with distributed mass parameters in mass matrix formulation along with simple matrix multiplication in matrix multiplication.

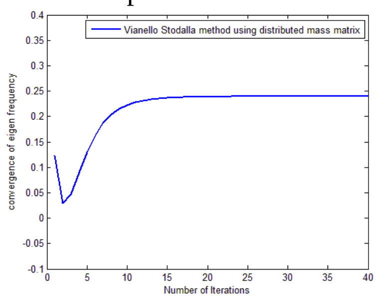

Figure.3. (a) is showing the Convergence of eigen frequency

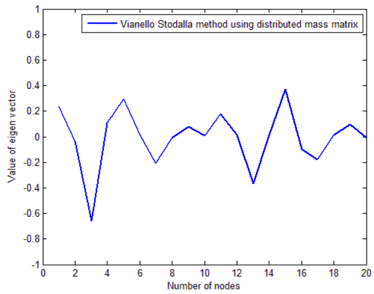

Figure.3. (b) Eigen frequency plot for each node 1-20

Using Inverse Power method: Inverse Power method for Eigen frequency comparison was conducted by taking the different combinations of lumped mass matrix and distributed mass matrix along with the simple matrix multiplication. A MATLAB program code was used for computations & its plots. The results of eigen frequency are plotted by different combinations are shown in Fig. 4-5

Fig. 4 is showing the program out come with lumped mass parameters in mass matrix formulation along with simple matrix multiplication.

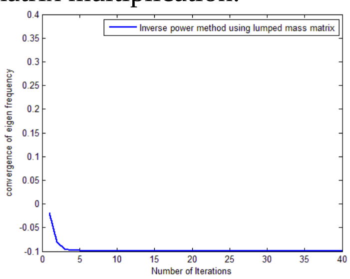

Figure.4. (a) Convergence of eigen frequency with nos. of iterative cycle

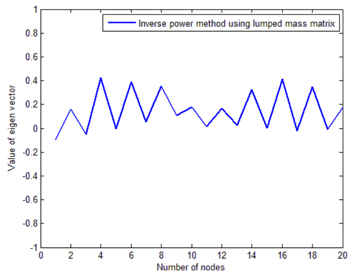

Figure.4. (b) Eigen frequency plot for each node 1-20

Fig.5 is showing the program out come with distributed mass parameters in mass matrix formulation along with simple matrix multiplication in matrix multiplication.

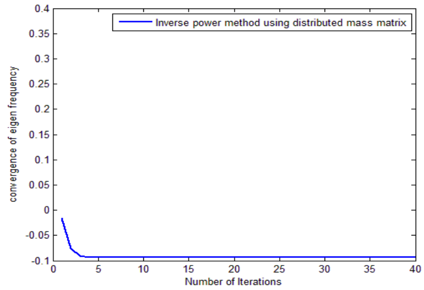

Figure.5. (a) Convergence of eigen frequency with nos. of iterative cycle.

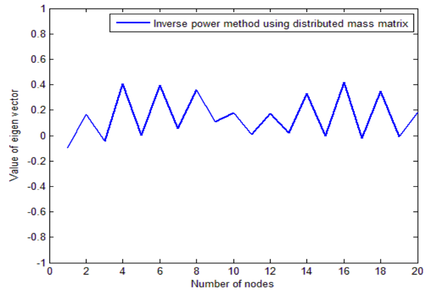

Figure.5. (b) Eigen frequency plot for each node 1-20

Using Rayleigh Ritz Iteration method: Rayleigh Ritz Iteration method for Eigen frequency comparison was conducted by taking the different combinations of lumped mass matrix and distributed mass matrix along with the simple matrix multiplication. A MATLAB program code was used for computations & its plots. The results of eigen frequency are plotted by different combinations are shown in Fig. 6-7

Fig.6 is showing the program out come with lumped mass parameters in mass matrix formulation with simple matrix multiplication in matrix multiplication part.

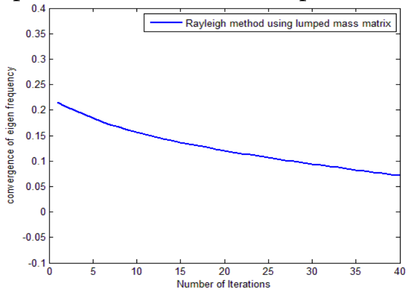

Figure.6. (a) Convergence of eigen frequency with nos. of iterative cycle

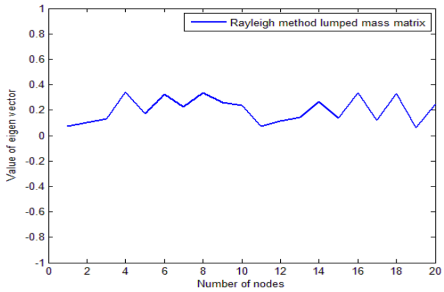

Figure.6. (b) Eigen frequency plot for each node 1-20

Fig.7 is showing the program out come with distributed mass parameters in mass matrix formulation with simple matrix multiplication in matrix multiplication part in matrix multiplication part.

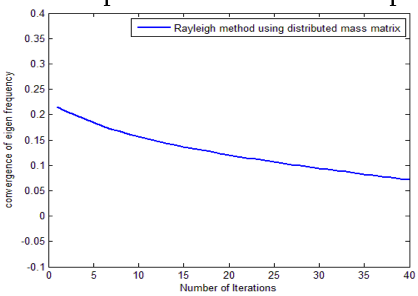

Figure.7. (a) Convergence of eigen frequency with nos. of iterative cycle

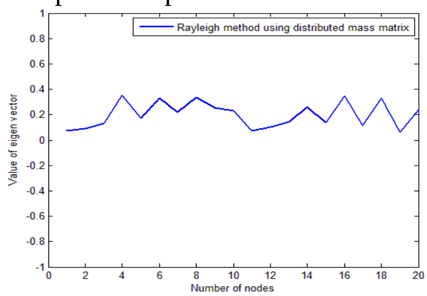

Figure.7. (b) Eigen frequency plot for each node 1-20

Using Modified Power method: Modified Power method for Eigen frequency comparison was conducted by taking the different combinations of lumped mass matrix and distributed mass matrix along with the simple matrix multiplication. A MATLAB program code was used for computations & its plots. The results of eigen frequency are plotted by different combinations are shown in Fig. 8-9

Fig.14 is showing the program out come with lumped mass parameters in mass matrix formulation along with simple matrix multiplication in matrix multiplication part.

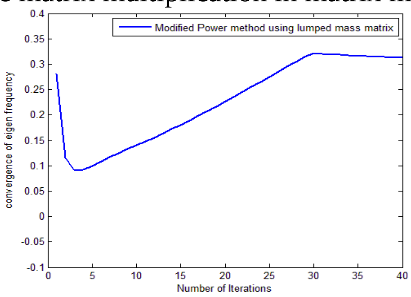

Figure.8. (a) Convergence of eigen frequency with nos. of iterative cycle

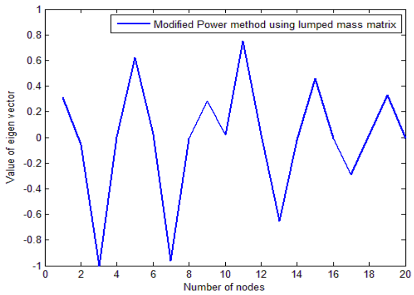

Figure.8. (b) Eigen frequency plot for each node 1-20

Fig.9 is showing the program out come with distributed mass parameters in mass matrix formulation along with simple matrix multiplication in matrix multiplication part.

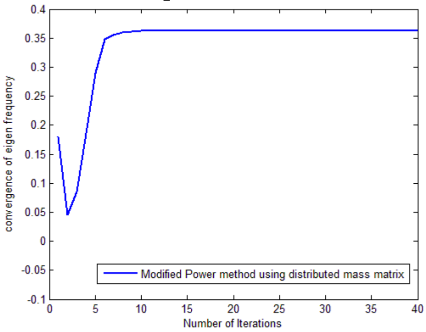

Figure.9. (a) Convergence of eigen frequency with nos. of iterative cycle

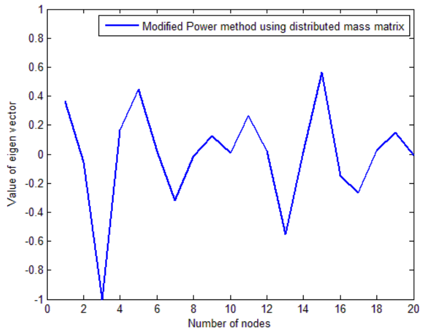

Figure. 9. (b) Eigen frequency plot for each node 1-20

A Comparative Study amongst the above four methods: Convergence of eigen frequency by taking lumped mass matrix method using the four methods mentioned above.

Convergence of eigen frequency by taking distributed mass matrix method using the four methods mentioned above.

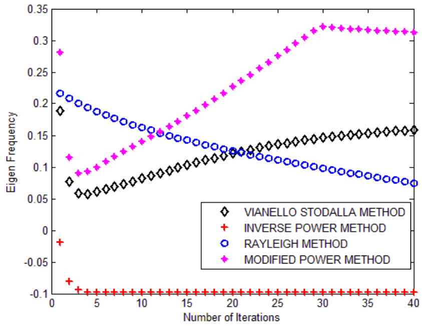

Figure.10. (a) Plot of the four methods considering lumped mass matrix

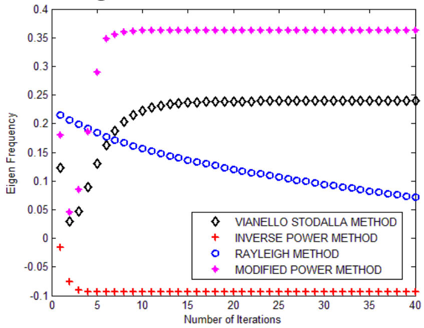

Figure.10. (b) Plot of the four methods by distributed mass

3. RESULTS AND DISCUSSION

The four different methods, namely the Vianello Stodalla method, inverse power method, Rayleigh quotient iteration method and modified power method have been employed on a common bridge truss model for the computation of eigen frequencies. The mass and the stiffness matrices have been computed by using the Finite element technique. Both lumped and distributed mass matrices were taken into consideration. The computation has been done by simple matrix multiplication. The graphs of the eigen frequencies have been plotted for the various combinations, from which the following inference can be drawn

- The Vianello stodalla method and the Modified power method give the Eigen frequency on the upper bound whereas the Inverse and Rayleigh methods give the frequencies on the lower bound side.
- The modified power method takes the minimum number of iterations to converge to the Eigen frequency whereas it is the Rayleigh method that takes the maximum number of iterations to converge to the Eigen frequency.
- By simply multiplying the matrices, the graphs obtained follow a similar pattern but are not able to converge the eigen values to a same point.
- From Table 1. It can be observed that the inverse power method converges the frequencies in the minimum number of iterations when compared to other methods.
- From Fig.10 (a) and Fig.10 (b) it can be said that the distributed mass matrix takes lesser no iterations to converge when compiled with any of the methods as compared to lumped mass matrix when compiled, for eigen frequency evaluation.

Table.1. Convergence Criteria for Eigen Frequency

Name of Method Applied	**Convergence using Lumped Mass**	**Convergence using Distributed Mass**
Vianello Stodalla Method	Converges after 40^{th} iteration.	Converges after 9^{th} iteration.
Inverse Power Method	Converges after 6^{th} iteration.	Converges after 5^{th} iteration.
Rayleigh Quotient Iteration Method	Converges after 77^{th} iteration.	Converges after 33^{rd} iteration.
Modified Power Method	Converges after 43^{rd} iteration.	Converges after 10^{th} iteration.

4. CONCLUSION

The characteristic polynomial uses the eigen values corresponding to zero, in many cases it means that it is giving the correct solution because it satisfying all the points from the above graph. The Rayleigh and the modified power method converge the frequencies close to the polynomial characteristic equation. Thus, we can conclude that these two methods converge the frequencies at a faster rate as compared to the other two methods discussed earlier.

It can also be concluded that the by incorporating the distributed mass matrix with any of the methods mentioned above the frequencies converge at a faster rate as when compared to lumped mass matrix. Also, that amongst all the four methods mentioned above Inverse Power method is able to converge the frequencies in the minimum number of iterations as compared to other methods.

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