A numerical study of fluid transport phenomenon over two Tandem and side-by-side circular cylinders by LBM

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ABSTRACT

This paper deals with the study of vortex dynamics and flow interference among two circular cylinders in side-by-side and tandem arrangement. It is recognized that, tandem and side-by-side arrange results are completely different from those observed on a single cylinder at the same Reynolds number. In the present work, the simulations are performed for a Reynolds number range varying from low to high Reynolds numbers, and the flow is solved by the mesoscopic Lattice Boltzmann method (LBM). The impact of the spacing among the cylinders is also considered. Streamline patterns and vorticity contours of flow across the cylinders and time histories are provided. It is seen that the present LBM approach produces results that are in excellent agreement with earlier numerical studies.

KEY WORDS: lattice boltzmann method, circular cylinder, tandem arrangement, side-by-side arrangement, d2q9 lattice model.

1. INTRODUCTION

Analysis of flow structures around arranged bluff bodies is a common problem in variety of engineering applications such as the flow past tall buildings, bridges and tube banks in heat exchanger, etc. (Fornberg, 1980). Cylindrical geometries often appear in engineering and industrial structures. There has been a wealth of interest in the rich topic of flow over a circular cylinder by von Karman. When flow takes place around a cylinder, over a range of Reynolds numbers, vortices are shed alternatively into the wake. Experimentalists, theoreticians and computational fluid dynamicists, including the lattice Boltzmann community, have sought to explain the interesting flow features associated with this benchmark problem in CFD. Only few researchers study the flow about tandem and side-by-side cylinders in the past. There is a number of ways to place cylinders in proximity to one another. In the present work, only combinations where one cylinder lies directly behind the other, tandem arrangement and side-by-side were considered. The vortex dynamics and flow interference when pairs of cylinders are placed in tandem arrangements is clearly described by Zdravkovich (1977).

Most of the published works concerning the flow around two cylinders were based on flow visualization in experiments. Numerical studies of flow around pairs of cylinders can also provide a better knowledge of the vortex dynamics, pressure distribution and fluid flows, in cases involving more complex arrangements (Meneghini, 2001). Ng and Ko (1995) numerically studied two cylinders in a tandem arrangement using a discrete vortex method. Mittal (1997) also studied incompressible unsteady flows past two cylinders in tandem arrangements. The wake structure can be determined from time-history measurements. From the literature, when two cylinders are arranged in tandem, the space between the cylinders has a strong influence on the formation and development of the wake of the second cylinder. The flow around two circular cylinders mainly depends on two governing parameters such as Reynolds number and non-dimensional spacing. The vortex dynamics is a three-dimensional phenomenon. However, 2-D simulations at different Reynolds number, as an approximation to the present problem, can be used to give the vortex dynamics in the wake and vortex impingement occurring when the cylinders are placed in a tandem and side-by-side configuration.

The flow about two cylinders has been the subject of many studies using conventional methods like finite element method, finite difference method and finite volume method in the past (Calhoun, 2002). In the last twenty years, Lattice Boltzmann Method (LBM) has used as a effective approach of CFD and it has achieved success in simulating fluid flows and heat transfer (Perumal & Dass, 2013). The LBM can be classified as a Lagrangian, explicit, hyperbolic approximation to the Navier-Stokes equation that has been derived within the framework of statistical mechanics. Being a relatively new and an effective method of flow computation, it is still being experimented with to realize its full scope of application and to test its ability to capture physics of the fluid-flow problems. Limited number of studies about flow around two circular cylinders using LBM can be found in the literature. Therefore, the two-dimensional LBM is used to investigate the vortex dynamics of two circular cylinders in tandem and side-by-side arrangement.

Lattice Boltzmann method: The discretized equation of single-relaxation-time LBM can be written as (Perumal & Dass, 2011)

$$f_i\left(\mathbf{x}+\mathbf{c}_i\Delta t, t+\Delta t\right)-f_i\left(\mathbf{x},t\right)=-\frac{1}{\tau}\left[f_i\left(\mathbf{x},t\right)-f_i^{eq}\left(\mathbf{x},t\right)\right] \quad (1)$$

Where $f_i(\mathbf{x},t)$ and $f_i^{eq}(\mathbf{x},t)$ are the particle and equilibrium distribution functions associated with discrete velocity $\boldsymbol{c_i}$ at ($\mathbf{x},t$). The *D2Q9* square lattice shown in Figure 1 used here has nine discrete velocities. For the *D2Q9* model the discrete velocity set $\{\boldsymbol{c_i}\}$ is written as (Perumal & Dass, 2013)

$$c_i = \begin{cases} (0,0), & i = 0; \\ c(\pm 1, 0), c(0, \pm 1), & i = 1,2,3,4; \\ c(\pm 1, \pm 1), & i = 5,6,7,8; \end{cases} \qquad (2)$$

The equilibrium particle distribution function for the *D2Q9* model is given by

$$f_i^{(eq)} = \rho w_i \left[1 + 3\left(\boldsymbol{c_i}.\boldsymbol{u}\right) + 4.5\left(\boldsymbol{c_i}.\boldsymbol{u}\right)^2 - 1.5(\boldsymbol{u}.\boldsymbol{u}) \right] \qquad (3)$$

and the lattice weights are $w_0 = 4/9$, $w_1 = w_2 = w_3 = w_4 = 1/9$, $w_5 = w_6 = w_7 = w_8 = 1/36$.

The macroscopic quantities such as density ρ and momentum density $\rho\boldsymbol{u}$ are defined in terms of the particle distribution function f_i as follows:

$$\rho = \sum_{i=0}^{N} f_i = \sum_{i=0}^{N} f_i^{eq} \qquad (4)$$

$$\rho\boldsymbol{u} = \sum_{i=0}^{N} f_i \boldsymbol{c_i} = \sum_{i=0}^{N} f_i^{eq} \boldsymbol{c_i} \qquad (5)$$

The relaxation time τ is related to the kinematic viscosity υ

$$\tau = \frac{6\upsilon + 1}{2} \qquad (6)$$

Where υ is the kinematic viscosity measured in lattice units.

The total resultant fluid force *F*, acting on a solid body by fluid can be written as (Perumal, 2014)

$$F = \sum_{all x_b} \sum_{\alpha=1}^{N_d} e_{\bar{\alpha}} \left[\tilde{f}_\alpha\left(x_b, t\right) + \tilde{f}_\alpha\left(x_b + e_{\bar{\alpha}}\delta t, t\right) \right] \times \left[1 - w\left(x_b + e_{\bar{\alpha}}\right) \right] \delta x / \delta t \qquad (7)$$

Where N_d is the number of non zero lattice velocity vectors and $w\left(x_b + e_{\bar{\alpha}}\right)$ is an indicator, which is 0 at x_f and 1 at x_b. The two most important characteristic quantities of flow around a cylinder are the coefficient of drag and coefficient of lift. Bounce-back boundary condition is used on the top and bottom walls. The coefficients are defined as (Perumal, 2014)

Coefficient of drag $$C_D = \frac{F_x}{1/2 \times \rho U_\alpha^2 D} \qquad (8)$$

Coefficient of lift $$C_L = \frac{F_y}{1/2 \times \rho U_\alpha^2 D} \qquad (9)$$

Where F_x and F_y are the *x*- and *y*- components of the total fluid force acting on the cylinder, *A* is the projected area. Boundary conditions plays important role in LBM simulations. In the present work, we used bounce-back boundary condition on the top and bottom walls, which indicate no-slip. Additional momentum to the inlet boundary proposed by Yu (2003) is used. At the outlet boundary, extrapolation boundary condition proposed by Guo (2002) is used. On the surface of the cylinder second order accurate boundary treatment proposed by Bouzidi (2001) is used. First, the validity of the LBM code is tested through comparisons with benchmark solutions for flow around a single circular cylinder in the next section.

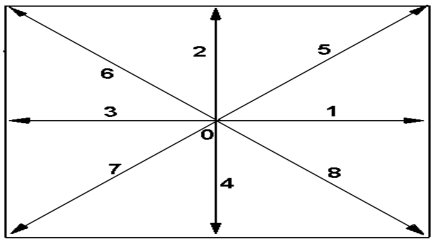


Fig.1. Two-Dimensional nine-velocity square lattice model.

Code validation – single circular cylinder: The flow past a single circular cylinder is studied as shown in Figure 2. Computations are carried out at *Re* = 60 for a blockage ratio *B* = *H*/*D* = 8 with the help of LBM (Height *H* = 80, Diameter *D* = 10). We present the coefficient of drag for different Reynolds numbers in Table 1. As the Reynolds number increases coefficient of drag (C_D) decreases. It is worth mentioning that the numerical simulations of our LBM are much closer to existing available results.

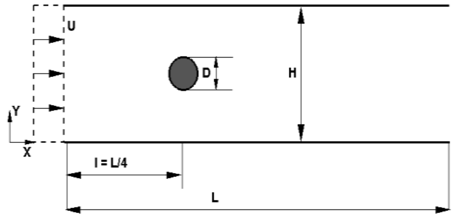


Fig.2. Schematic diagram of the flow past a circular cylinder.

Table.1. Coefficent of Drag (C_D) for different circular cylinder steady-flow Reynolds numbers.

Authors	*Re*=10	*Re*=20	*Re*=30	*Re*=40
Fornberg (1980)	-	2.00	-	1.50
Calhoun (2002)	-	2.19	-	1.62
Single cylinder - LBM present	3.21	2.25	1.74	1.50

Two circular cylinders – Tandem: In order to study the proximity effect on vortex shedding, simulations done for the case of two circular cylinders of same diameter in tandem configuration. First, the low Reynolds number range from 20 to 100 is considered. In the present tandem configuration 500×80 lattice size is used. This lattice size has been finalized after conducting grid independence study. It is known that, the use of high dense lattice size leads to exponential increase in computational time. It is known that, when the central distance of the circular cylinders is small, the fluid flow among two cylinders remains steady and laminar, while in the second circular cylinder wake is unsteady. It is seen that, central gap of the circular cylinders increases, the wake of the second cylinder changes from unsteady to steady gradually. Therefore, the distance among the two cylinders is chosen as a moderate value of 3D. Figure 3-5 shows the vorticity contours for low Reynolds number *Re* = 20, 40 and 60 respectively. It is observed that the secondary circular cylinder does not allow the vortex of the primary circular cylinder to develop. From the figures, it is seen that in tandem arrangement, the flow is symmetric and uniform in the primary cylinder and the pattern is similar to the case of single circular cylinder.

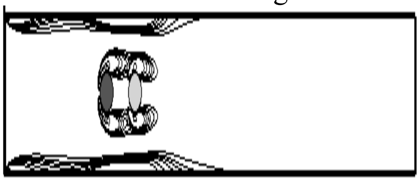


Fig.3. Vorticity contours of tandem arrangement at *Re* = 20.

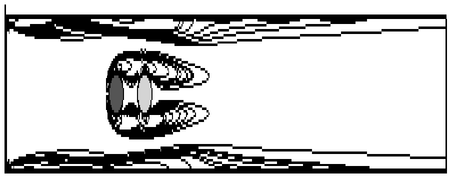


Fig.4. Vorticity contours of tandem arrangement at *Re* = 40.

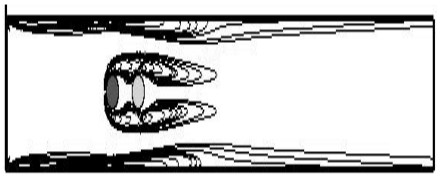
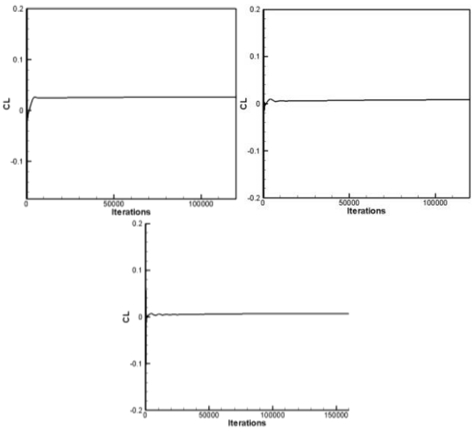


Fig.5. Vorticity contours of tandem arrangement at *Re* = 60.



(a) *Re* = 20 (b) *Re* = 40 (c) *Re* = 60

Fig.6. Time dependent lift coefficients C_L at (a) *Re* = 20, (b) *Re*= 40 and (c) *Re*= 60.

Figure 6 shows the time dependent lift coefficients C_L at different Reynolds numbers such as *Re* = 20, 40 and 60 respectively. In the present work, special focus is paid to the impact of the distance among the cylinders. Therefore, moderate Reynolds number of *Re* = 100 is chosen. The gap between the centres of the cylinders, which have the same diameter, is chosen in range of 1.5D to 4.0D. Figure 7-10 shows the vorticity contours at *Re* = 100 for spacing between circular cylinders 1.5D, 2.0D, 3.0D and 4.0D. It is observed that at *Re* = 100, the secondary cylinder does not allow the vortex of the primary cylinder to develop. It is also seen that, vortices shed only from the secondary cylinder. From the Figures 7-10, one can see that the circular cylinders act as a single body, with only one vortex wake forming behind the secondary cylinder. The separating shear layer from the primary cylinders involves the secondary body. The interaction between these shear layers takes place only in the base region of the secondary cylinder, with a consequent vortex formation and shedding occurring behind this body. Figure 11-14 depicts the instantaneous streamline patterns for a gap of 1.5D, 2.0D, 3.0D and 4.0D at *Re* = 100. Alternate eddies are formed in between the cylinders and in the endside of the secondary cylinder. The size of the eddy in between the cylinder is smaller when compared to the endside of the secondary cylinder. Frequency of vortex shedding is found to decrease with the introduction of secondary cylinder in any side of the primary cylinder.

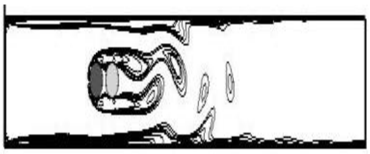


Fig.7. Vorticity contours of cylinders at *Re* = 100 (1.5D)

Fig.8. Vorticity contours of cylinders at *Re* = 100 (2.0D)

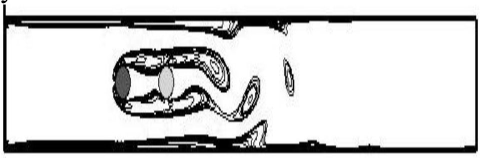


Fig.9. Vorticity contours of cylinders at *Re* = 100 (3.0D)

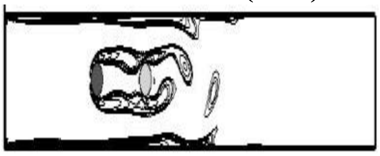


Fig.10. Vorticity contours of cylinders at *Re* = 100 (4.0D)

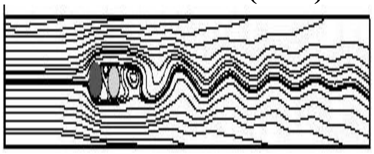


Fig.11. Instantaneous streamline patterns at *Re* = 100. (1.5D)

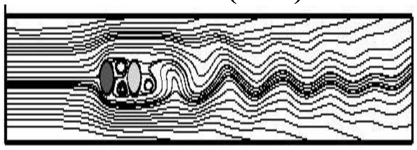


Fig.12. Instantaneous streamline patterns at *Re* = 100. (2.0D)

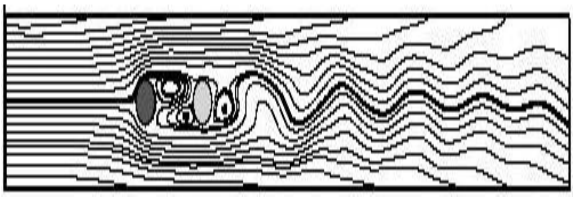


Fig.13. Instantaneous streamline patterns at *Re* = 100. (3.0D)

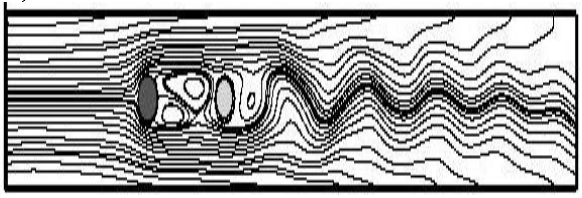
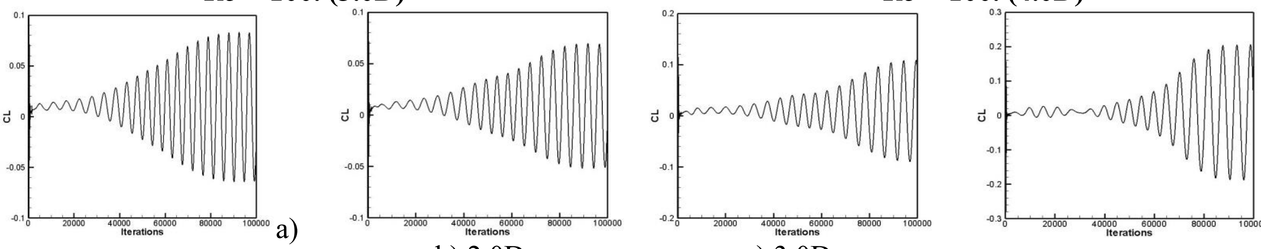


Fig.14. Instantaneous streamline patterns at *Re* = 100. (4.0D)



a) 1.5D b) 2.0D c) 3.0D d) 4.0D

Fig.15. Time dependent lift coefficients at *Re* = 100 on a lattice size of 500×80.

From Figure 11 and 12, it is observed that the wake of primary cylinder is seen to reattach to secondary cylinder. A symmetrical and uniform flow pattern is seen between the two circular cylinders. The analysis of time dependent lift coefficients for the cases of distances 1.5D, 2.0D, 3.0D and 4.0D at *Re* = 100 is shown in Figure 15. Though results are not presented in this section, we have observed that the presented results are agrees well with other researchers (Sharman, 2001; Calhoun, 2002).

Two circular cylinders – Side by Side: Next to study the proximity effect on vortex dynamics, simulations done for the case of two cylinders of same diameter in side-by-side arrangement. Both the cylinders are chosen as same diameter. The Reynolds number range from 30 to 100 is considered. The 500×80 lattice size is used for the side-by-side configuration. Figure 16-18 shows the vorticity contours for low Reynolds number *Re* = 30, 50 and 100 respectively. From the figures it is observed that the side-by-side circular cylinder results are completely different from tandem arrangement. Both the cylinders create vortex shedding. At low Reynolds numbers the flow is uniform and symmetric for both the top and bottom wall cylinders. As the Reynolds number increases it is seen that the effect of vortex shedding more on the bottom wall cylinder. Figure 19-21 depict the instantaneous streamline patterns for different Reynolds numbers 30, 50 and 100 respectively. At low Reynolds number the flow is symmetric with respect to centre line of the channel. It is observed that, there is repulsive force acting on the side-by-side cylinders. A pressure drop also occurs as the fluid flows through the gap of cylinders. It is seen that, the effect of upstream and downstream results of side-by-side arrangement of circular cylinders is different from tandem arrangements.

Fig.16. Vorticity contours of two circular cylinders at *Re* = 30.

Fig.17. Vorticity contours of two circular cylinders at *Re* = 50.

Fig.18. Vorticity contours of two circular cylinders at *Re* =100

Fig.19. Instantaneous streamline patterns at *Re* = 30.

Fig.20. Instantaneous streamline patterns at *Re* = 50.

Fig.21. Instantaneous streamline patterns at *Re* = 100.

2. CONCLUSIONS

In the present work, the flow past single and two circular cylinders in tandem and side-by-side arrangement are studied using LBM. Streamline patterns and vorticity contours are presented for different Reynolds numbers. Special focus is paid on the distance among two cylinders. This problem serves two purposes: it demonstrates that (i) LBM can be used to handle curved boundaries (ii) LBM has the ability to capture the unsteady flow states characterized by the circular cylinders at relatively higher Reynolds number. It is shown that present LBM, as a credible alternative to the continuum-based methods, holds very good promise in CFD. The present study also concludes that the LBM can capture the flow bahaviour of the circular cylinder in tandem and side-by-side configuration and is a very good tool for complex geometry studies. Our future work is to study the three-dimensional flow behavior of two circular cylinders.

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