Computational Study on Heat Transfer from a Shrouded Upright Rectangular Fin Array: A Typical Case Study

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ABSTRACT

A computational study is made to analyse the mixed convective heat transfer from a shrouded upright rectangular fin array. A typical case is chosen by fixing the Grashof number and clearance at 2.18×10^5 and 0.10 respectively. Variations of bulk fluid temperature, local Nusselt number with axial direction are reported. Overall Nusselt number values are noted. Behaviours of temperature and velocity profiles are also observed.

KEY WORDS: shrouded fin array; clearance; bulk fluid temperature; local Nusselt number; overall Nusselt number.

1. INTRODUCTION

Augmentation of heat transfer through fin surface is a common practice for engineering applications such as electronic cooling systems, automobile reactors, heat exchangers, space vehicles etc. In addition, it is very much helpful in rapid heat removal processes. So, the topic of heat transfer through fin surfaces is a prime interest for the researchers. Enhancement of heat transfer under natural convection by placing a shroud in front of the fin-tips was first computationally revealed by the article of Karki and Patankar (1987). Later, the report (Giri, 2003) has investigated the natural convective heat and mass transfer from shrouded vertical fin array computationally. A mathematical formulation of heat and mass transfer over a shrouded vertical fin array is developed in this report. Recently published articles (Giri and Das, 2012; Das and Giri, 2014; Pathak and Giri, 2017) have investigated different problems through mixed convective heat transfer using shrouded vertical fin array. The article (Giri, 2015) has studied combined heat and mass transfer over shrouded vertical fin array. As shrouded vertical fin array problem is a central topic of interest for many researchers, so, examining one of its cases would be interesting. In the present work, a modified computational code is developed by following the above mentioned articles to investigate a particular case of a shrouded upright (vertical) fin array under mixed convection.

Analysis: Fig.1a. shows the physical setup selected for the present investigation. It consists of a set of rectangular plate fins having thickness 't_f', length 'L' and height 'H' are attached to a vertical base plate maintain a temperature of 'T_w' (100^0C). The surrounding temperature is chosen as 'T_0' (20^0C). An adiabatic shroud is placed in front of the fin array such a way that a clearance 't_c' is maintained between the fin tip and the shroud. Non-dimensional clearance 't_c^*'is fixed to 0.10 (3mm) in the present analysis. Inter-fin spacing 'S' is maintained between two consecutive fins. '$S+t_f$' is considered to be the fin pitch. The coordinate system is chosen such a way that along *y*- direction fin height is considered and along *z*- axis fin length is considered. *x*- direction represents base or shroud length. Thus, assembly of base, fin array and shroud consist series of identical rectangular channels. Each channel has a dimension of $L\times(S\times(H+t_c)+t_f\times t_c)$. The computational domain is shown in the Fig. 1b. The computational domain is shown in the Fig. 1b. As all channels are identical, present study is limited to only one such identical channel. The mathematical formulation is defined for the present analysis as follows:

Figure.1. Physical setup

2. MATHEMATICAL FORMULATION

Governing equations and boundary conditions: Conservation of mass, momentum and energy equations are used to model the present problem as follows:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

Where, along x-, y- and z-direction, u, v, w velocities are selected.

Momentum, energy and fin conduction equations are expressed by the following equation.

$$\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}\right)=\gamma\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\alpha+\beta \quad (2)$$

θ represents velocities and temperature in case of momentum equations and energy/fin conduction equation respectively. Pressure gradient and viscous dissipation in momentum equations and energy equation are represented by α. In the present analysis velocities encounter is very small and hence ignored the viscous dissipation term. β represents buoyancy in flow direction.

Following No-slip condition, velocities at the wall along the tangential direction are supposed to be zero. In order to satisfy the impermeability condition normal velocities are considered zero. Flow is considered as laminar throughout the investigation. For this reason, inlet velocities are considered carefully such that flow always fall within the limits of laminar flow and it depends on the hydraulic diameter of the problem. The gradient of temperature in normal direction is considered zero as the shroud is considered adiabatic.

Local Nusselt number (Nu_l): In order to find the heat transfer at every axial location (along *z*-axis), a parameter called local Nusselt number is defined. Due to the excess temperature between base and the bulk fluid local heat transfer is expected to prevail. Local heat transfer coefficient in non-dimensional form is termed as local Nusselt number and is defined as

$$Nu_l=\frac{-\int_0^H \left.\frac{\partial T}{\partial x}\right|_{x=0.5t_f} dy+\int_0^H \left.\frac{\partial T}{\partial x}\right|_{x=S+0.5t_f} dy-\int_0^{0.5t_f} \left.\frac{\partial T}{\partial y}\right|_{y=H} dx-\int_{0.5t_f}^{S+0.5t_f} \left.\frac{\partial T}{\partial y}\right|_{y=0} dx-\int_{S+0.5t_f}^{S+t_f} \left.\frac{\partial T}{\partial y}\right|_{y=H} dx}{(H+H+S+t_f)(T_w-T_b)}. \quad (3)$$

Overall Nusselt number: Overall heat transfer coefficient in non-dimensional form termed as overall Nusselt number of the fin-base system and it is defined based on the temperature difference of base and surroundings, and is written as

$$Nu_O=\frac{\int_0^L\left(-\int_0^H \left.\frac{\partial T}{\partial x}\right|_{x=0.5t_f} dy+\int_0^H \left.\frac{\partial T}{\partial x}\right|_{x=S+0.5t_f} dy-\int_0^{0.5t_f} \left.\frac{\partial T}{\partial y}\right|_{y=H} dx-\int_{0.5t_f}^{S+0.5t_f} \left.\frac{\partial T}{\partial y}\right|_{y=0} dx-\int_{S+0.5t_f}^{S+t_f} \left.\frac{\partial T}{\partial y}\right|_{y=H} dx\right)dz}{L(H+H+S+t_f)(T_w-T_0)}. \quad (4)$$

Computational procedure: Eqs.(1-2) are the basic governing equations for the present problem. These governing equations are discretised using finite volume approach. For this purpose algorithm used is semi implicit method for pressure linked equation revised, commonly known as SIMPLER algorithm. The method of this computational process is clearly explained in (Patankar, 1980). Computational study requires grid sensitivity test. To test the sensitivity of the grids, numerical runs are performed by using grids of 14×36, 16×40, and 18×46 for S^*= 0.2, another grid combinations of 20×36, 22×40 and 24×46 for S^* = 0.3 and 24×36, 26×40 and 28×46 for S^*= 0.4 are deployed in the *x-y* plane. Nominal deviation of overall results is observed for the selected grid combinations. Present results are attained with a combination of grids of 16×40 for S^*= 0.2, 22×40 for S^* = 0.3 and 26×40 for S^* = 0.4. In the *z*-direction 112 grids are used to cover the entire fin length. In order to check the precision of the results, the experimental results (Maughan and Incropera, 1990) are reproduced through the present code. This shows a favourable agreement. Comparison of the results is shown in the Table.1.

Table.1. Experimental validation

Rayleigh Number	(Maughan and Incropera, 1990) (Nu_o)	Present Work (Nu_o)
3500	5.85	6.35
7000	6.1	6.40
13500	6.8	7.10

3. RESULTS AND DISCUSSION

Axial variation of local Nusselt number (Nu_l) and Bulk fluid temperature (θ_b): Local Nusselt number (Nu_l) and Bulk fluid temperature (θ_b) variation with axial direction is shown in the Fig.2a-b. This variation is plotted for selected non-dimensional fin spacing of 0.2 & 0.4. Inlet velocities ($W_{in,mix}$) are chosen as 1071 and 1318. Bulk fluid temperature (θ_b) is a direct indicator of sensible heat removed from the fluid or it represents the total heat absorbed by fluid during the heat transfer process. On the other hand, the definition of local Nusselt number (Nu_l) is based on the excess temperature between base and the bulk fluid. From Fig.2a, it can be seen that bulk fluid temperature rises very sharply near the entry region because of the sharp thermal boundary layer. After certain length of the fin, the incremental rate of bulk fluid temperature reduces due to the reduction of the thermal gradient which can be understood from the temperature profiles which will be described in the following section. The larger the fin spacing the lower is the attainment of bulk fluid temperature value for the selected inlet velocities. The primary reason behind this is the significant effect of buoyancy in case of the larger fin spacing. Broader duct space will result higher buoyancy effect relative to the smaller one. Near inlet, heat transfer rate rapidly warms up the fluid and reducing its heat absorbing capacity gradually. That is why the variation of local Nusselt number shows a sharp monotonic decrease near the entrance and it attains a fully developed value at the exit. Effect of inlet velocities in case of local

Nusselt number is negligible but for the lower inlet velocities bulk fluid temperature predicts higher value than that of the higher inlet velocities.

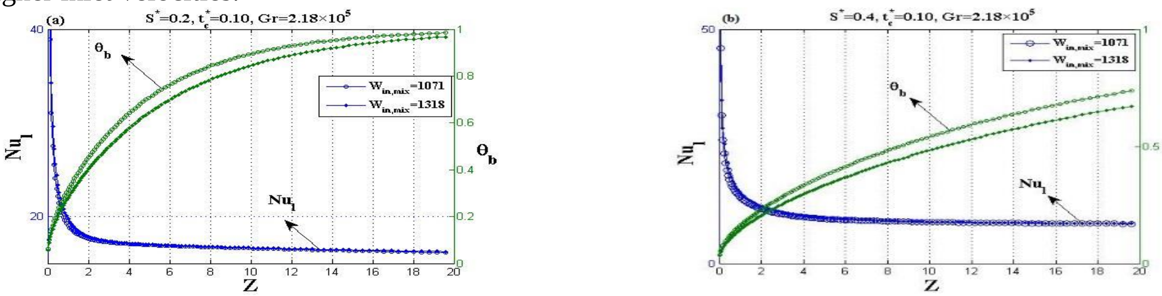

Figure.2. Axial variation of local Nusselt number and Bulk fluid temperature

Temperature profile: The dimensionless temperature profile is depicted in the Fig. 3a-b. Temperature profile is defined such a way that higher value means lower dimensional temperature and vice versa. It is plotted for the axial locations Z=0.5356 (inlet section) and Z=19.38 (outlet section). Fig.3a shows the fluid temperature profile very near the entrance region for the fin spacing S^*=0.2. Parabolic distribution of temperature profile is noticed along X-direction. Fluid temperature near base along Y direction adopts the temperature of the base thereafter progressively takes a constant value throughout the fin length and the temperature reduces to the temperature of the fluid. This indicates the completion of the creation of thermal boundary layer. Clearance region is unheated at the entrance. The fluid temperature profiles at outlet section are depicted in the Fig.3b. At outlet (Z=19.38), in the inter fin region fluid becomes saturated to nearly the wall temperature.

Figure.3. Axial development of temperature profile

Velocity profile: Axial development of the velocity profile is sketched in Fig.4a-b. It is plotted at the same locations as that are plotted for the temperature profile. Fig.4a. illustrate the development of the velocity profile at inlet section (Z=0.5356) for S^*=0.2. Nearly parabolic W-velocity profile is seen along X-direction. Through the fin height (i.e., along-Y), velocity assumes a zero value at the base and increases to a persistent value. It is also observed that the velocity profile at the clearance takes parabolic shape for all the selected fin spacing. At outlet (Fig.4b, Z=19.38), the velocity in the clearance region is reduced as compared to that obtained in the inlet section (Z=0.5356). Velocity increases at the inter fin region at outlet section (Z=19.38) due to the higher bypass of the fluid through the larger fin spacing. It is also important to mention that sharp fall of velocity in the clearance region is due to the enhancement of the fluid velocity over the fin base system due to the larger flow bypass.

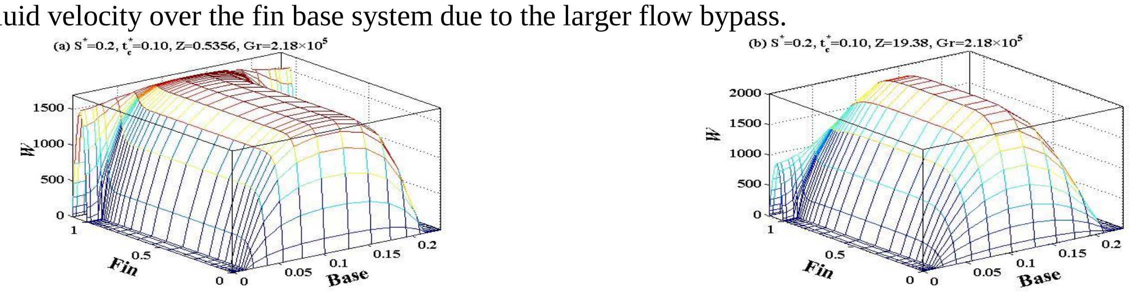

Figure.4. Development of *W*- velocity profile

***Overall Nusselt number* (Nu_o):** Overall Nusselt numbers for the selected parameters are shown in the Table.2.

Table.2. Overall Nusselt number (Nu_o)

S^*	t_c^*	Gr	$^{**}W_{in,mix}$	Nu_o
0.2	0.10	2.18×10^5	1071	0.40359E+01
0.2	0.10	2.18×10^5	1318	0.48821E+01
0.3	0.10	2.18×10^5	1071	0.51426E+01
0.3	0.10	2.18×10^5	1318	0.59478E+01
0.4	0.10	2.18×10^5	1071	0.55095E+01
0.4	0.10	2.18×10^5	1318	0.61818E+01

** Non-dimensional velocities are assumed in such a way that flow remains laminar.

4. CONCLUSIONS

A numerical experiment is made to analyse the mixed convective heat transfer from a shrouded upright rectangular fin array for a particular case of Gr=2.18 × 10^5 and t_c^*=0.10. Results indicate that bulk fluid temperature near inlet rises very sharply due to the sharp thermal boundary layer. The reduction of the thermal gradient reduces the increasing rate of the bulk fluid temperature. Local Nusselt number shows a sharp decrease near the inlet and it attains a fully developed value at the exit. Overall Nusselt number is tabulated. Evolutions of temperature and velocity profiles are reported.

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