APPLICATION OF FORMING LIMIT DIAGRAM AND YIELD SURFACE DIAGRAM TO STUDY ANISOTROPIC MECHANICAL PROPERTIES OF ANNEALED AND UNANNEALED SPRC 440E STEELS

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ABSTRACT

Sheet metal forming is a vital operation in the manufacturing of a number of automobile components. The quality of the formed component depends largely on the formability of the sheet metal. In the present study, the formability of SPRC440E steel sheets by deep drawing have been characterized and the forming limit diagram (FLD’s) have been experimentally determined by conducting punch stretching experiments on unannealed and annealed SPRC440E sheets using a double acting hydraulic press. Formability observed from FLD’s has been correlated with microstructure, mechanical properties and formability parameters. From the experimental results, it has been found that the sheets have a favorable grain size and mechanical properties to meet the requirements of good formability for annealed SPRC 440E steel. It is also confirmed by Erichsen cupping test. It was also observed that metal stretchability increases with increasing value of strain hardening exponent (n) and drawability increases with increasing value of normal anisotropy (r_n) value. It is well known that FLD and anisotropic properties of a substance sensitively depends upon its yield surface diagram. Therefore a suitable yield criterion is chosen to describe the plastic behavior and predict the nature of FLD of SPRC440E. Here, the yield criterion chosen is Hill's 1948 yield criterion.

Keywords: Sheet metal Forming, Erichsen Cupping Test, Annealing, Forming Limit Diagram, Hill's 1948 Yield Criterion and Yield Surface Diagram.

INTRODUCTION

Sheet metal forming is an important manufacturing process in which a sheet of metal is plastically deformed into desired shape of a component without defects. The quality of formed component largely depends on the formability of sheet metal. The formability of a sheet metal is defined as the ability of metal to undergo deformation without fracture or wrinkling. The amount of strain that a sheet metal can tolerate just before localized failure is called limit strain. The limits of formability in sheet metal operations are described in terms of the principal strain histories and strain distributions as a tool for determining forming limits. Goodwin used a combination of cup- and tension-tests to obtain a failure band in both the negative and positive quadrants of minor strain, creating the general form of the forming-limit diagram. In order to reveal the effects of planar anisotropy on formability, Marciniak et al. performed an experimental study involving steel, aluminum, and copper.

The concept of the forming-limit diagram introduced by Keeler is now widely used as one of the criteria for optimizing stamping processes and in the designing of dies. Forming limit diagrams can be regarded as material property curves and industrially the curve for a particular material is always established experimentally. Local thinning, splits, tears and wrinkles cannot be accepted when forming automotive panels. Forming limit diagrams (FLDs) have been determined experimentally by conducting punch-stretching experiments for aluminum-killed extra-deep drawing (EDD) low carbon steel sheets.

The effect of n and r on the strain distribution characteristics has been analyzed. Earing tendency during drawing is expected to be high due to its high planar anisotropy. Sheet metal forming is a widely utilized manufacturing process in the automotive, aeronautical and consumer goods industries. The manufacturers of automobile prefer steel for favorable mechanical properties such as high strength, ductility and toughness. It is also demanded for vehicle safety and crashing strength of the vehicle Forming Limit Diagrams were evaluated for the Interstitial Free (IF) steels and the tensile properties and formability parameters were correlated with the FLD. Formability of extra-deep drawing (EDD) low carbon steel sheets have been determined experimentally by conducting punch-stretching experiments.
Formability, observed from FLDs, has been correlated with microstructure, mechanical properties and formability parameters like strain hardening coefficient (n) and normal anisotropy ($r_m$). In more complex sheet forming operations, multiaxial stresses and strains are imposed. The limiting dome height (LDH) test, in which a clamped sheet is deformed by a hemispherical punch (out-of-plane stretching) until fracture, provides a useful measure of the forming limit for many punch stretching operations of the sort used in manufacture of automotive vehicles.

**Need for Study**

Current trends in automotive sector demands the requirement of steel by deep drawing in various applications with variable complexity in design and manufacturing, which in turn demands for variability in formability and drawing characteristics from low to high. Recently developed SPRC 440E alloy steels offer a very good combination of strength and ductility. Extensive uses of SPRC 440E alloy sheets also used away from boundaries of automotive sector are in the field of household application. This investigation was carried out for a better understanding of the forming behavior of SPRC 440E alloy steel by means of the experimental determination of Hill’s 1948 criterion: Theoretical Background

The Hill’s 1948 criterion for anisotropic materials for a plane stress condition is expressed by the following equation:

$$\sigma_1^2 - 2r_0(\sigma_1\sigma_2) + r_0(1+r_0)\sigma_2^2 = \sigma_0^2$$

Where $\sigma_0/\sigma_{90} = [(r_{90}/(1+r_0))/r_{90}(1+r_0)]^{1/2}$.

It should be noted that in above formula is used for the case when principle directions of stress tensor coincide with the principal anisotropic axes.

This yield equation denotes an ellipse in $\sigma_1$-$\sigma_2$ plane. The points on the boundary of the ellipse depict the plastic state, while the points inside the curve correspond to the state of elastic stress. The points outside the curve have no physical meaning.

This criterion was chosen for SPRC440E because of many reasons. Its basic assumptions are easy to understand and manipulate. The parameters it uses have direct physical meaning. Also, this criterion needs smaller no. of mechanical parameters for determining the yield function. Besides, it has proven to be in good agreement with the experimental values for steel alloys as it predicts well the variation of $r$-values with $\theta$ (angle made by the longitudinal direction of the test specimen with rolling direction). The theoretical yield locus obtained is also in good approximation with the experimental locus.

FLD is related to yield surface diagram as:

- Let's consider $\psi$ as the angle between the line joining origin to the plane strain point on yield surface and the line to the point denoting initial yield surface determined from the loading path.
- The formability of a material depends upon the ease with which this angle is swept from the initial strain path. Sharper the curvature of the yield surface diagram, more easily the deformation state will switch to plane stress state which means lower limiting strain. Thus formability will be less.

**EXPERIMENTAL PROCEDURE**

The material was investigated under unannealed and annealed conditions. The detail of the experimental work is described below.

The specimens were cut from a rolled sheet of thickness 0.65mm along the three directions namely longitudinal ($0^\circ$), diagonal ($45^\circ$), and transverse ($90^\circ$) to the rolling directions of the sheet.
Fig 2. Rectangular Tension Test Specimens with Reduced Parallel Section

The specimens were grouped into two to conduct tests under (i) Unannealed and (ii) Annealed conditions. The specimens were solution annealed at a temperature of 1373° for one hour followed by furnace cooling. The specimens were cleaned free from scaling.

Study of Chemical composition and grain size: The chemical composition of SPRC material has been analyzed as per ASTM A751 – 11 standards by an emission spectrometer. The observed composition of SPRC was tabulated. The grain structure was observed by polishing the sheet with fine emery of grades 4'O', 3'O', 2'O', 1'O' at rotation of 90° for each polishing after which it was disc polished in a velvet roller with additives such as diamond paste with Hifin spray by same methodology of 90° rotation. Then 5% concentrated Nital was added, washed after 2 min and allowed to dry for 5 min in an air blower. The grain structure was observed in a 3-Eye optical Microscope of 100X magnification.

Tensile and Formability Parameters: The most widely used intrinsic test of sheet metal formability is the uniaxial test. Specimens were prepared as per ASTM E 517 standard using specimen of 12.5 mm wide with a gauge length of 50mm. Specimens were gripped at both ends and pulled at a constant rate at 400KN in a universal testing machine (UTM). All tests were done at room temperature and the extensions were recorded using an extensometer. For r determination

Fig 3. Axes used in defining normal and planar anisotropy

Fig 4. Stretching experiment hemispherical punch setup

The strain hardening exponent (n), the plastic strain ratio (r) and the planar anisotropy (Δr) are the conventional indicators of formability of sheet metals. The strain hardening exponent ‘n’ was found as per ASTM E646 using empirical representation over the range of interest of the true-stress versus true-strain curve.

\[
\sigma = k \varepsilon^n
\]

Where, \(\sigma\) - True stress
\(k\) - Strength coefficient
\(n\) - Strain hardening exponent
\(\varepsilon\) - True strain

The plastic strain ratio (r) is the ratio of true width strain to true thickness strain and the value of ‘r’ is evaluated as per ASTM E517

\[
r = \frac{\varepsilon_w}{\varepsilon_t}
\]

Where, \(\varepsilon_w\) - Width strain, and
\(\varepsilon_t\) - Thickness strain.
\(\varepsilon_w = \ln \left( \frac{w_f}{w_0} \right)\) and
\(\varepsilon_t = \ln \left( \frac{t_f}{t_0} \right)\)
Where, \( w_t \) - Change in width \\
\( t_t \) - Change in thickness \\
\( w_0 \) - original width \\
\( t_{0} \) - original thickness

The \( r_m \) - weighted average of \( r \) values obtained in three directions: 0° (parallel), 45° (diagonal), and 90° (transverse) to the rolling direction and planar anisotropy (earing tendency) \( \Delta r \) are calculated as follows,

\[
\begin{align*}
    r_m &= \frac{r_{00}^0 + 2r_{45}^0 + r_{900}^0}{4} \\
    \Delta r &= \frac{r_{00}^0 + r_{900}^0 - 2r_{45}^0}{2}
\end{align*}
\]

Where,

\( r_m \) - Average plastic strain ratio (or) normal plastic anisotropy \\
\( r_{00}^0 \) - Longitudinal to rolling direction \\
\( r_{45}^0 \) - 45° to rolling direction \\
\( r_{900}^0 \) – Transverse to rolling direction

**Hemispherical punch stretching test:** The Hecker’s forming limit diagram methodology has been evaluated in SPRC 440E sheet for unannealed and annealed condition. This evaluation involves three steps- Grid making, Punch stretching and Strain measurements. The punch stretching experiment is carried out on the specimen using 100 ton closed frame hydraulic press machine of 500 mm stroke length and working height 800 mm. A set of specimens with a fixed length 100 mm and different widths of 100, 90, 70, 60, 50, 40, 30, 20 mm were prepared. Decreasing the width from 100 mm to 20 mm causes change in the state of strain from near balanced-biaxial tensile through plane strain to uniaxial tension. Circular grids of 5 mm dia were electrochemically etched on the entire specimen. The specimen was held in a circular lock bead and stretched to failure using a 100 mm diameter punch.

As a result of stretching, circular grids deformed in to elliptical shape. These ellipses have major and minor strain axes. The major strain (\( e_1 \)) is always defined to be the direction in which the greatest positive strain has occurred without regard to original blank edges or the sheet rolling direction. The minor strain (\( e_2 \)) is defined to be 90° to the major strain direction. The major strain (\( e_1 \)) is always positive while the minor strain (\( e_2 \)) may be zero, positive or negative. The maximum (\( e_1 \)) and associated (\( e_2 \)) values measured in critical areas on the formed part are plotted on the graph containing the forming limit curve by locating the point of intersection of the (\( e_1 \)) and (\( e_2 \)) strains. The strains were obtained by measurement of major and minor axis using Mylar tape. The circles affected by fracture are considered as “failures” while others are considered as safe.

![Fig 5. Schematic diagram of a typical tool setup for FLD](image)

**Erichsen Cupping Test:** The SPRC440E sheets were investigated under Erichsen cupping test to check the strength, hardness, anisotropy and strain hardening rate of material considering drawing, redrawing and reverse redrawing phenomenon of sheet metal. Erichsen Model 100 Sheet Metal Cupping Tester was used with 30 KN drawing force and 10 KN blank holder force was used with 18 mm stroke length as per ASTM 643-84, the SPRC sheet was clamped between two ring dies of Erichsen tester and a ball was force against the sheet until it fractured and the depth of the bulge before fracture was measured using vernier. The observed values were tabulated for unannealed and annealed SPRC specimens.

**RESULT AND DISCUSSION**

The chemical composition, micro structure, tensile property, Erichsen cupping values and formability parameters for SPRC440E sheet specimen were determined by the above said procedure. The observations were tabulated for discussion.

**Chemical Composition:** The Chemical Composition of the SPRC440E sheets is as shown in table 1.
Here the silicon acts as a good deoxidizing agent which kills the metal oxide formation, carbon attributes to hardness and strength of the specimen which will be changed to spheriodized structure while annealing, that favors the ductile nature of the steel. The variation in material alloying from nominal which has been added with 0.14 % of magnesium is added to improve the hot working characteristics, 0.003 to 0.005 % of phosphorus has been added to improve strength and corrosive resistance since its application in automotive and household has the exposure towards air and water which induces corrosion and 0.003 to 0.005 % of Niobium (Nb) is alloyed to stabilize carbon atom while annealing process is carried on the specimen. The presence of Nb increases the drawability.

Grain Structure: The grain structure SPRC440E sheet has been observed under optical microscope for unannealed and annealed specimen. The microstructure examination of Unannealed SPRC 440E revealed step structures with dispersed particles of carbide in the austenitic matrix with twinned grains as shown in Fig 6 & 7. The grain sizes are measured in fixed magnification are found to be 7-8 for Unannealed and 5-6 for annealed which decreases after annealing that favors formability since fine grains are stronger and have low strain hardening exponent as that of coarse grains. Grain size of steel sheets influences formability in different ways. Fine grain steels are quite strong, but they have low strain hardening exponent and limited formability. Coarse grain steels have better formability. Grain sizes of ASTM 7 or 8 are usually a good compromise between formability and surface appearance.

Table 1. Chemical Composition of SPRC440E

<table>
<thead>
<tr>
<th>Alloy</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>T-Al</th>
<th>S-Al</th>
<th>Nb</th>
<th>Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPRC 440E</td>
<td>0.022</td>
<td>0.022</td>
<td>0.14</td>
<td>0.070</td>
<td>0.091</td>
<td>0.017</td>
<td>&lt;0.005</td>
<td>Balance</td>
</tr>
</tbody>
</table>

The graphite flakes in the unannealed condition which similar to flakes in grey cast iron cause high stress concentration which lowers the elongation nature of the SPRC 440E. These after annealing process converted into graphite balls similar to ductile cast iron that enhances the ductility in the material which favors elongation and there by improves formability characteristics.

Tensile properties: The tensile properties of the unannealed & annealed SPRC 440E were determined by tensile test at room temperature with the specimen axis oriented at 0°, 45° and 90° of the rolling direction and reported in Table 2 & 3. The strain hardening exponent (n) value increases around an average of 0.04 for all the three direction of the rolling for SPRC 440E annealed conditions than that of unannealed condition. It is observed that the increase in (n) value increases the stretchability of the material. The stretchability of SPRC 440E is more for annealed than that of unannealed condition.
It is observed the average strength coefficient \( k \) 1215 Mpa for unannealed specimen and 1040 Mpa for annealed specimen.

The yield strength and the ultimate tensile strength of SPRC 440E are reduced by annealing process around 50Mpa and 100 Mpa respectively which increases the formability nature of SPRC 440E specimen. The ultimate tensile strength of SPRC annealed specimens is found to be 356Mpa and 667Mpa. The decrease in yield and ultimate tensile strength improves the formability nature of SPRC 440E.

**Formability parameters:** The formability parameters of unanneled and annealed SPRC 440E were determined from tensile test are tabulated in Table 4 & 5. The plastic strain ratio (\( r \)) value is doubled after annealing compared to the unannealed SPRC 440E in all the specimens irrespective of the rolling direction of the sheet. This indicates that annealing process improves drawability of the sheets. If the (\( r_m \)) value is high then the metal has high drawability since it shows good resistance to thinning in the thickness direction during forming. The product nr is indicative of overall performance factor this value is lesser in annealed specimen compared to unannealed condition. It shows stretchability is better in annealed condition. The tendency of earing is related to the planar anisotropy (\( \Delta r \)). The planar anisotropy is a measure of the amount of high points or ears that will develop on the edges of the deep drawing of the cylindrical cups. For most applications values of (\( \Delta r \)) near zero are preferred because such values imply a minimal tendency to form tears when metals are drawn in to cylindrical cups. The planar anisotropy values obtained in the present study are -0.01and -0.018 for Unannealed and annealed conditions respectively. It shows that SPRC 440E sheet is lesser tendencies to earing. The sheets can be used for drawing operations with and without annealing.

### Table 4. Formability Parameters of Unannealed SPRC 440E

<table>
<thead>
<tr>
<th>Orientation relative to rolling direction</th>
<th>SPRC 440E Unannealed conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r )</td>
<td>( nr )</td>
</tr>
<tr>
<td>0°</td>
<td>0.598</td>
<td>0.1794</td>
</tr>
<tr>
<td>45°</td>
<td>0.609</td>
<td>0.1418</td>
</tr>
<tr>
<td>90°</td>
<td>0.60</td>
<td>0.138</td>
</tr>
<tr>
<td>Average *</td>
<td>0.604</td>
<td>0.152</td>
</tr>
</tbody>
</table>

### Table 5. Formability Parameters of Annealed SPRC 440E

<table>
<thead>
<tr>
<th>Orientation relative to rolling direction</th>
<th>SPRC 440E Annealed conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r )</td>
<td>( nr )</td>
</tr>
<tr>
<td>0°</td>
<td>1.12</td>
<td>0.3808</td>
</tr>
<tr>
<td>45°</td>
<td>1.332</td>
<td>0.3836</td>
</tr>
<tr>
<td>90°</td>
<td>1.18</td>
<td>0.3044</td>
</tr>
<tr>
<td>Average *</td>
<td>1.241</td>
<td>0.3562</td>
</tr>
</tbody>
</table>

\*Average=(x_{0} + 2 x_{45} + x_{90})/4 where x is \( r \)-value and \( nr \)-value, 

\( \Delta r = (r_0 -2 r_{45} + r_{90})/2 \)

**Erichsen Cupping Test:** In that cupping test the Erichsen no is more (13.7) in annealed than unannealed SPRC 440E Sheet (12.4) are reported in table 6 & 7, it is found specimen from this experiment it is found that SPRC 440E favorable for sheet metal work.

### Table 6. Erichsen Cupping Value of SPRC 440E Unannealed condition

<table>
<thead>
<tr>
<th>Nature of plates</th>
<th>Trail 1</th>
<th>Trail 2</th>
<th>Trail 3</th>
<th>Trail 4</th>
<th>Trail 5</th>
<th>Trail 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Plates</td>
<td>12.41</td>
<td>12.49</td>
<td>12.42</td>
<td>12.44</td>
<td>12.48</td>
<td>12.49</td>
</tr>
<tr>
<td>Long Plate</td>
<td>12.46</td>
<td>12.47</td>
<td>12.46</td>
<td>12.46</td>
<td>12.47</td>
<td>12.46</td>
</tr>
</tbody>
</table>

### Table 7. Erichsen Cupping Value of SPRC 440E annealed condition

<table>
<thead>
<tr>
<th>Nature of plates</th>
<th>Trail 1</th>
<th>Trail 2</th>
<th>Trail 3</th>
<th>Trail 4</th>
<th>Trail 5</th>
<th>Trail 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Plates</td>
<td>13.69</td>
<td>13.72</td>
<td>13.7</td>
<td>13.74</td>
<td>13.7</td>
<td>13.72</td>
</tr>
<tr>
<td>Long Plate</td>
<td>13.7</td>
<td>13.72</td>
<td>13.7</td>
<td>13.7</td>
<td>13.69</td>
<td>13.7</td>
</tr>
</tbody>
</table>

**Experimental prediction of forming limit diagram:** FLDs are evaluated experimentally for unannealed and annealed SPRC 440E of thickness 0.65 Figs 8 & 9 show the experimental FLDs for unannealed and annealed SPRC 440E of 0.65mm thickness respectively. These FLDs were drawn by considering the fractured, safe and onset necked regions. The necked points are joined by using linear polynomial for drawing region and second order polynomial for stretching region.
Comparing the fig 8 & fig 9, the limiting major strain at unannealed condition is 0.32 and 0.39 for annealed condition. It shows the annealing process improves the formability of SPRC 440 E steel. The improvement in formability is also observed in tension-tension zone and tension compression zone.

**Yield Surface Diagram**
- The values of $r_0$, $r_{45}$, $r_{90}$, $\sigma_0$, and $\sigma_{90}$ were obtained experimentally for annealed and unannealed state of SPRC440E.
- Using the yield function discussed in section 1.2, the yield surface diagram was plotted in the plane of $\sigma_1/y$ and $\sigma_2/y$, where ‘y’ is the reference yield stress.
- The obtained graph is shown in figure 10.

The graph shown in the figure depicts the yield surface in the first quadrant i.e. the tension-tension zone. As we can see, for higher value of ‘r’, the graph elongates along the biaxial stress axis. The reference stress chosen here was the uniaxial yield stress along the rolling direction.

**CONCLUSION**

In the present study an attempt has been made to determine the forming limit diagram (FLD) and yield surface diagram of unannealed and annealed SPRC 440 E of thickness 0.65mm. FLDS were experimentally evaluated for the above mentioned materials using hemispherical punch stretching test. Effect of chemical composition present in the material was determined. The yield surface diagram was determined with help of Hill's 1948 criterion using the experimental calculated parameters. It shows well the variation of yield surface diagram for annealed and unannealed state having different r-values. It also shows the dependence of their FLDS on yield surface diagram. The following conclusions are made from the study.

1. The micro examination of SPRC 440E steel alloys in the annealed and unannealed conditions revealed that the effect of annealing improves the formability of SPRC 440E alloys. Micro examination of annealed SPRC 440E alloys revealed equiaxed grains of size ranging “between” 7-8, the grain sizes of the alloys were found to decrease after annealing between 5-6.
2. The tensile properties of unannealed and annealed SPRC 440E steels were determined and compared. Effect of annealing is found to decrease the tensile strength and yield strength of SPRC 440E alloys. Annealing improves the strain hardening exponent (n) of SPRC 440E alloys from 0.249 to 0.294.

3. The formability parameters of SPRC 440E alloys were determined. The Effect of annealing improves the \( r_m \) from 0.604 to 1.241.

4. The yield surface diagram also shows the elongation of yield surface along biaxial for annealed state which explains its greater formability on account of its increased anisotropic ratios. It also explains the lower value of limiting strain for unannealed state.

5. Tendency of earing is related to the planar anisotropy (\( \Delta r \)). The \( \Delta r \) value is the measure of amount of high points or ears that will develop on the edges of the deep drawing of the cups. To minimize the earing defect, \( \Delta r \) value nearer to zero is preferred. The value of \( \Delta r \) for specimen is found

6. Tensile properties and formability parameters were found to be in good correlation with the formability of the sheets. It was observed that metal stretchability increases with increasing value of strain hardening exponent (n) and drawability increases with increasing value of normal anisotropy \( r_m \) value.

7. The Erichsen value for the Unannealed SPRC 440E (12.4) and annealed SPRC 440E (13.7) is found to more which enhance the formability of SPRC 440E.

8. Variation of r-values: The elongation of yield surface for annealed state along the biaxial stress axis suggests r-values are greater than those for unannealed state which is also evident from the experimental r-values given in Table 4 and Table 5. This means annealed SPRC440E has better formability.

9. Effect on limiting strain: For higher values of r, \( \phi \) tends to be lower due to sharp curvature near the biaxial region. Thus with decrease in r-values, limiting strain should increase, but the unannealed SPRC440E has lesser limiting strain than its annealed state. It has been found that materials with r-values less than 1 show abrupt decrease in limiting strain. It could be explained in terms of sharpness of yield locus near biaxial stress axis. This sharp curvature allows unannealed SPRC440E to quickly approach plane thus results in relatively low limiting strains.

REFERENCES


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